Určete definiční obory funkcí:

1) $f(x) = \sqrt{\frac{x-2}{3+x}} - \ln(x+2)$
2) $f(x) = \ln(x^2 + x - 6)$
3) $f(x) = \sqrt{x^2 - x - 12} + \ln\frac{2x-4}{-x+3}$
4) $f(x) = \sqrt{\frac{2-3x}{x-1}} + \log_2(-x^2 + 5x)$
5) $f(x) = \frac{2}{-3+x} + \sin x - \sqrt{x^2 + 5x + 4}$
6) $f(x) = \sqrt{\frac{2x+3}{4x-1}} - 3\log_{\frac{1}{2}}(-x^2 + x + 6)$
7) $f(x) = \frac{\ln(x-3)}{\sqrt{4x-5}}$
8) $f(x) = arctg(x) - \arcsin\frac{x-1}{2}$
9) $f(x) = \ln x - \sqrt{-x^2 + 2x + 35}$

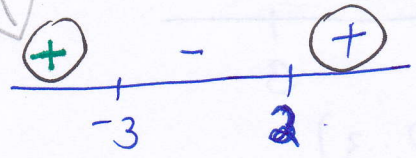
Určete definiční obory:

1) $f(x) = \sqrt{\frac{x-2}{3+x}} - \ln(x+2)$

I. $\frac{(+)x-2}{3(+)x} \geq 0$

$x-2=0 \quad 3+x=0$

$x=2 \quad x=-3$



$(-\infty; -3) \cup \langle 2; +\infty)$

-3 tam nepatří kvůli podmínce ze zlomku

2 tam patří, protože bylo $\frac{x-2}{3+x} \geq 0$ doleinoma se

II. $x+2>0$

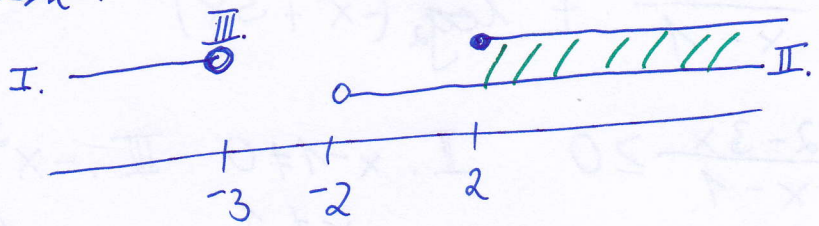
$x>-2$

$(-2; +\infty)$

III. zlomek: $3+x \neq 0$

$x \neq -3$

Průnik řešení:



$Df = \langle 2; +\infty)$

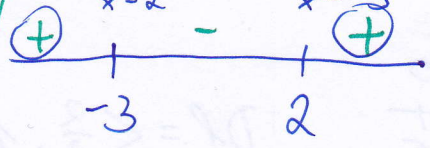
2) $y = \ln(x^2 + x - 6)$

$(+)x^2 + x - 6 > 0$

$(x-2)(x+3) > 0$

$x-2=0 \quad x+3=0$

$x=2 \quad x=-3$



$Df = (-\infty; -3) \cup (2; +\infty)$

→ rozklad: $-2 \cdot 3 = -6$

$-2+3 = +1$

$x^2 (+1)x (-6)$

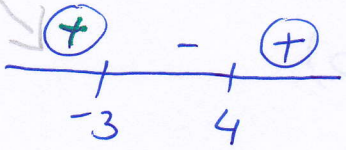
3) $y = \sqrt{x^2 - x - 12} + \ln \frac{2x-4}{-x+3}$

I. $+x^2 - x - 12 \geq 0$

$(x-4)(x+3) \geq 0$

$x-4=0 \quad x+3=0$

$x=4 \quad x=-3$



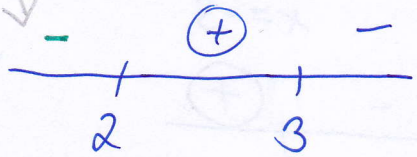
$(-\infty; -3\rangle \cup \langle 4; +\infty)$

II. $\frac{+2x-4}{-x+3} > 0$

$2x-4=0 \quad -x+3=0$

$2x=4 \quad 3=x$

$x=2$

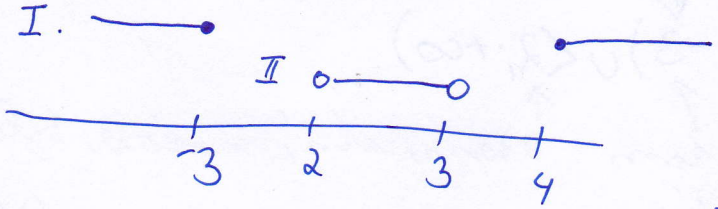


$(2; 3)$

III. $-x+3 \neq 0$

$x \neq +3$

Průnik:



nemá žádný průnik $\Rightarrow D_f = \emptyset$ prázdná množina

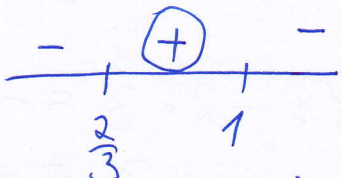
4) $y = \sqrt{\frac{2-3x}{x-1}} + \log_2(-x^2+5x)$

I. $\frac{2-3x}{x-1} \geq 0$

$2-3x=0 \quad x-1=0$

$2=3x \quad x=1$

$\frac{2}{3}=x$



$\langle \frac{2}{3}; 1)$

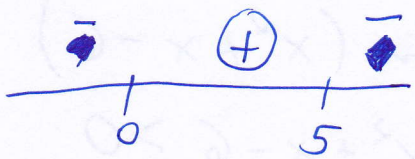
II. $x-1 \neq 0$

$x \neq 1$

III. $-x^2+5x > 0$

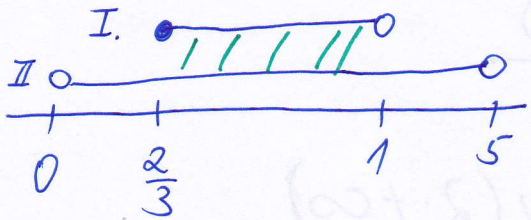
$-x \cdot (x-5) > 0$

$x=0 \quad x=5$



$(0; 5)$

Průnik:



$D_f = \langle \frac{2}{3}, 1)$

5) $y = \frac{2}{-3+x} + \sin x - \sqrt{x^2+5x+4}$

I. $-3+x \neq 0$

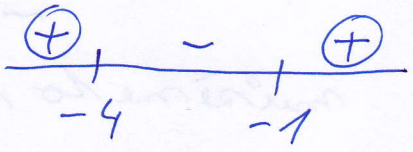
$+3 \neq x$

II. $x^2+5x+4 \geq 0$

$(x+4)(x+1) \geq 0$

$x+4=0 \quad x+1=0$

$x=-4 \quad x=-1$



$(-\infty; -4\rangle \cup \langle -1; +\infty)$

Průnik:

$Df = (-\infty; -4\rangle \cup \langle -1; 3) \cup (3; +\infty)$

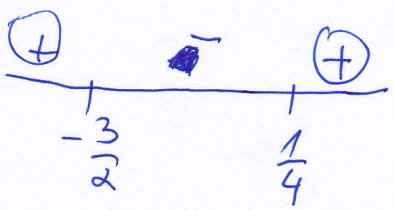
6) $y = \sqrt{\frac{2x+3}{4x-1}} - 3 \cdot \log_{\frac{1}{2}}(-x^2+x+6)$

I. $\frac{2x+3}{4x-1} \geq 0$

$2x+3=0 \quad 4x-1=0$

$2x=-3 \quad 4x=1$

$x=-\frac{3}{2} \quad x=\frac{1}{4}$



$(-\infty; -\frac{3}{2}\rangle \cup (\frac{1}{4}; +\infty)$

II. $4x-1 \neq 0$

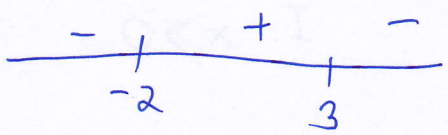
$x \neq \frac{1}{4}$

III. $-x^2+x+6 > 0$

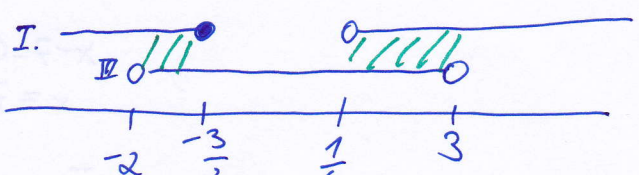
$-(x^2-x-6) > 0$

$-(x-3)(x+2) > 0$

$x=3 \quad x=-2$



$(-2; 3)$



Průnik:

$Df = (-2; -\frac{3}{2}\rangle \cup (\frac{1}{4}; 3)$

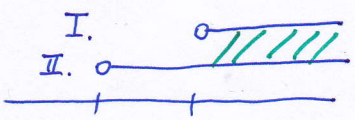
7) $y = \frac{\ln(x-3)}{\sqrt{4x-5}}$

I. $x-3 > 0 \rightarrow x > 3 \quad (3; \infty)$

II. $4x-5 > 0$

$4x > 5$

$x > \frac{5}{4}$



$Df = (3; +\infty)$

musí být rovnítko
protože je odmocnina ve jmenovateli

8) $y = \operatorname{arcotg} x - \arcsin \frac{x-1}{2}$

$\operatorname{arcotg} x \rightarrow$ má $D_f = R$, tedy zde není podmínka

$\arcsin x \rightarrow$ má $D_f = \langle -1; 1 \rangle \rightarrow$ tak z toho plyne podmínka:

$$-1 \leq \frac{x-1}{2} \leq 1$$

musíme to vyřešit zvlášť:

$-1 \leq \frac{x-1}{2} \quad / \cdot 2$

$-2 \leq x-1 \quad / +1$

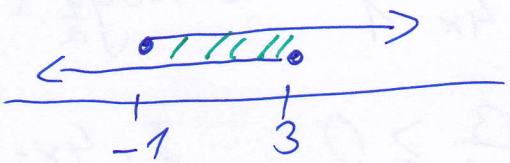
$-1 \leq x$

$\frac{x-1}{2} \leq 1 \quad / \cdot 2$

$x-1 \leq 2 \quad / +1$

$x \leq 3$

Průnik:



$D_f = \langle -1; 3 \rangle$

9) $y = \ln x - \sqrt{-x^2+2x+35}$

I. $x > 0$

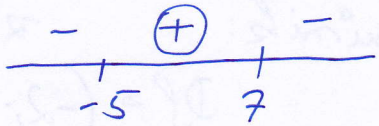
II. $-x^2+2x+35 \geq 0$

$-(x^2-2x-35) \geq 0$

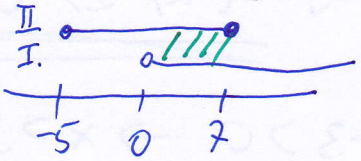
$-(x-7)(x+5) \geq 0$

$x-7=0 \quad x+5=0$

$x=7 \quad x=-5$



Průnik:



$D_f = (0; 7\rangle$