Najděte inverzní funkci : (určete $D_f, H_f, D_{f^{-1}}, H_{f^{-1}}$)

1) $y = 2^{x+4} - 5$

2) $y = \dfrac{3x-1}{2x+4}$

3) $y = \ln(x+4) - 2$

4) $y = e^{x-3} + 1$

5) $y = \log_3(x-1) + 2$

6) $y = \sin\left(x + \frac{\pi}{2}\right) - 1$

7) $y = \arccos(x+2) - \pi$

8) $y = 2 \cdot \operatorname{tg}(x - \pi) + 2$

9) $y = -\operatorname{arccotg}\left(x + \frac{2}{3}\right) - \pi$

10) $y = \cos\left(x - \frac{\pi}{2}\right) + 4$

11) $y = \operatorname{cotg}(x + \pi)$

1) $x = 2^{y+4} - 5$ $\qquad D_f = \mathbb{R} = H_{f^{-1}}$

$x + 5 = 2^{y+4}$

$\log_2(x+5) = y + 4$ $\rightarrow$ $x + 5 > 0$, $x > -5$

$\boxed{\log_2(x+5) - 4 = y}$ $\qquad D_{f^{-1}} = (-5; \infty) = H_f$

2) $x = \dfrac{3y - 1}{2y + 4} \quad / \cdot (2y+4)$ $\qquad D_f = \mathbb{R} - \{-2\} = H_{f^{-1}}$

$2x + 4 \neq 0$

$x \neq -2$

$2xy + 4x = 3y - 1$

$2xy - 3y = -1 - 4x$ $\qquad D_{f^{-1}} = \mathbb{R} - \{\frac{3}{2}\} = H_f$

$y(2x - 3) = -1 - 4x$

$2x - 3 \neq 0$

$\boxed{y = \dfrac{-1 - 4x}{2x - 3}}$

$x \neq \frac{3}{2}$

3) $x = \ln(y + 4) - 2$ $\qquad x + 4 > 0$, $x > -4$

$x + 2 = \ln(y + 4)$ $\qquad D_f = (-4, \infty) = H_{f^{-1}}$

$e^{x+2} = y + 4$

$\boxed{e^{x+2} - 4 = y}$ $\qquad D_{f^{-1}} = \mathbb{R} = H_f$

4) $x = e^{y-3} + 1$ $\qquad D_f = \mathbb{R} = H_{f^{-1}}$

$x - 1 = e^{y-3}$

$x - 1 > 0$

$\ln(x - 1) = y - 3$ $\qquad x > 1$

$\boxed{\ln(x - 1) + 3 = y}$ $\qquad D_{f^{-1}} = (1, \infty) = H_f$

5) $x = \log_3(y-1) + 2$

$x - 2 = \log_3(y-1)$

$3^{x-2} = y - 1$

$\boxed{3^{x-2} + 1 = y}$

$x - 1 > 0$

$x > 1$

$Df = (1, \infty) = Hf^{-1}$

$Df^{-1} = R = Hf$

6) $x = \sin\left(y + \frac{\pi}{2}\right) - 1$

$x + 1 = \sin\left(y + \frac{\pi}{2}\right)$

$\arcsin(x+1) = y + \frac{\pi}{2}$

$\boxed{\arcsin(x+1) - \frac{\pi}{2} = y}$

$-\frac{\pi}{2} \leq x + \frac{\pi}{2} \leq \frac{\pi}{2}$ (aby byl sinus prostá funkce)

$-\pi \leq x \leq 0$

$Df = \langle -\pi; 0 \rangle = Hf^{-1}$

$-1 \leq x + 1 \leq 1$

$-2 \leq x \leq 0$

$Df^{-1} = \langle -2; 0 \rangle = Hf$

7) $x = \arccos(y+2) - \pi$

$x + \pi = \arccos(y+2)$

$\cos(x + \pi) = y + 2$

$\boxed{\cos(x+\pi) - 2 = y}$

$-1 \leq x + 2 \leq 1$

$-3 \leq x \leq -1$

$Df = \langle -3; -1 \rangle = Hf^{-1}$

$0 \leq x + \pi \leq \pi$ (aby byl cosinus prostý)

$-\pi \leq x \leq 0$

$Df^{-1} = \langle -\pi; 0 \rangle = Hf$

8) $x = 2\,\mathrm{tg}(y - \pi) + 2$

$x - 2 = 2\,\mathrm{tg}(y - \pi) \quad |:2$

$\frac{x-2}{2} = \mathrm{tg}(y - \pi)$

$\boxed{\mathrm{arctg}\,\frac{x-2}{2} + \pi = y}$

$-\frac{\pi}{2} < x - \pi < \frac{\pi}{2}$ (aby byl tg prostý)

$\frac{\pi}{2} < x < \frac{3}{2}\pi$

$Df = \left(\frac{\pi}{2}; \frac{3}{2}\pi\right) = Hf^{-1}$

$Df^{-1} = R = Hf$

9) $x = -\operatorname{arccotg}\left(y+\frac{2}{3}\right)-\pi$ $\qquad D_f = R = H_{f^{-1}}$

$x+\pi = -\operatorname{arccotg}\left(y+\frac{2}{3}\right) \quad /\cdot(-1)$

$-x-\pi = \operatorname{arccotg}\left(y+\frac{2}{3}\right)$

$\operatorname{cotg}(-x-\pi) = y+\frac{2}{3}$

$\boxed{\operatorname{cotg}(-x-\pi)-\frac{2}{3} = y}$

aby byl cotg posl.:

$0 < -x-\pi < \pi$

$\pi < -x < 2\pi \quad /\cdot(-1)$

$-\pi > x > -2\pi$

$D_{f^{-1}} = (-2\pi, -\pi) = H_f$

10) $x = \cos\left(y-\frac{\pi}{2}\right)+4$

$x-4 = \cos\left(y-\frac{\pi}{2}\right)$

$\arccos(x-4) = y-\frac{\pi}{2}$

$\boxed{\arccos(x-4)+\frac{\pi}{2} = y}$

$0 \le x-\frac{\pi}{2} \le \pi$

$\frac{\pi}{2} \le x \le \frac{3}{2}\pi$

$D_f = \langle \frac{\pi}{2}; \frac{3}{2}\pi \rangle = H_{f^{-1}}$

$-1 \le x-4 \le 1$

$3 \le x \le 5$

$D_{f^{-1}} = \langle 3; 5 \rangle = H_f$

11) $x = \operatorname{cotg}(y+\pi)$

$\operatorname{arccotg} x = y+\pi$

$\boxed{\operatorname{arccotg} x - \pi = y}$

$0 < x+\pi < \pi$

$-\pi < x < 0$

$D_f = (-\pi, 0) = H_{f^{-1}}$

$D_{f^{-1}} = R = H_f$