LIMITY

1) $\lim\limits_{n\to\infty}\left(n^2+2n+5\right)$

2) $\lim\limits_{n\to\infty}\left(n^3-2n+5\right)$

3) $\lim\limits_{n\to\infty}\left(\dfrac{4}{n+1}-n\right)$

4) $\lim\limits_{n\to\infty}\dfrac{2n^3+5n-1}{4n^3+2n-5}$

5) $\lim\limits_{n\to\infty}\dfrac{2n^2+5n}{n^4-2n+1}$

6) $\lim\limits_{n\to\infty}\dfrac{3n^3-2n+1}{-2n^2+n+5}$

7) $\lim\limits_{n\to\infty}\dfrac{-n^3-5n+1}{-2n+2}$

8) $\lim\limits_{n\to\infty}\dfrac{5n-4}{2n+2}$

9) $\lim\limits_{n\to\infty}\dfrac{n^2+5n-1}{4n^5-2n+2}$

10) $\lim\limits_{n\to\infty}\dfrac{2^n+3^n}{4^n-2^n}$

11) $\lim\limits_{n\to\infty}\dfrac{3^n-2\cdot5^n+1}{4^n+3\cdot2^n}$

12) $\lim\limits_{n\to\infty}\dfrac{2^{2n}+3^{3n}}{3^{2n}-2^{3n}}$

13) $\lim\limits_{n\to\infty}\left(2^n-4^n+5^n\right)$

14) $\lim\limits_{n\to\infty}\dfrac{5\cdot3^{2n}-2^n}{3\cdot2^n-9^n}$

15) $\lim\limits_{n\to\infty}\dfrac{n!}{(n-1)!}$

16) $\lim\limits_{n\to\infty}\dfrac{(n+3)!}{(n+4)!}$

17) $\lim\limits_{n\to\infty}\dfrac{n!+(n-1)!}{(n+1)!}$

18) $\lim\limits_{n\to\infty}\dfrac{(n-2)!+(n-1)!}{(n+1)!}$

19) $\lim\limits_{n\to\infty}\dfrac{n!+(n+2)!}{(n+1)!}$

20) $\lim\limits_{n\to\infty}\dfrac{n!-(n-1)!}{(n-1)!}$

21) $\lim\limits_{n\to\infty}\dfrac{n!+(n-1)!}{(n+1)!-n!}$

22) $\lim\limits_{n\to\infty}\left(1+\dfrac{4}{n}\right)^2$

23) $\lim\limits_{n\to\infty}\left(1+\dfrac{3}{n}\right)^n$

24) $\lim\limits_{n\to\infty}\left(\dfrac{n-4}{n}\right)^n$

25) $\lim\limits_{n\to\infty}\left(\dfrac{n}{n-2}\right)^n$

26) $\lim\limits_{n\to\infty}\left(\dfrac{2n}{2n+4}\right)^n$

27) $\lim\limits_{n\to\infty}\left(\dfrac{n-3}{n+1}\right)^n$

28) $\lim\limits_{n\to\infty}\left(\dfrac{2n+1}{n-2}\right)^n$

29) $\lim\limits_{n\to\infty}\left(\dfrac{3n-2}{4n+12}\right)^n$

30) $\lim\limits_{n\to\infty}\left(\dfrac{4n-1}{3n+1}\right)^n$

31) $\lim\limits_{n\to\infty}\left(\dfrac{2n+3}{2n-2}\right)^n$

32) $\lim\limits_{n\to\infty}\sqrt{\dfrac{n-3}{n^2+1}}$

33) $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^4-2x}}{\sqrt{x^2+2x-1}}$

34) $\lim\limits_{n\to\infty}\sqrt[n]{2n^4}$

35) $\lim\limits_{n\to\infty}\sqrt[n]{n^3-2n+1}$

36) $\lim\limits_{n\to\infty}\sqrt[n]{5n^3}$

37) $\lim\limits_{n\to\infty}\sqrt[n]{2n+4}$

38) $\lim\limits_{n\to\infty}\sqrt{n^2+2n}+n$

39) $\lim\limits_{n\to\infty}\sqrt{n^2+2n}-n$

40) $\lim\limits_{n\to\infty}\sqrt{n^2+1}-n$

41) $\lim\limits_{n\to\infty}\sqrt{n+3}-\sqrt{n-1}$

42) $\lim\limits_{x\to3}\dfrac{x^2+3}{x+3}$

43) $\lim\limits_{x\to3}\dfrac{x^2-9}{x-3}$

44) $\lim\limits_{x\to3}\dfrac{x^2-6x+9}{81-x^4}$

45) $\lim\limits_{x\to3}\dfrac{x^2+2x-15}{x^2-8x+15}$

46) $\lim\limits_{x\to-2}\dfrac{x^2-x-6}{x^3+6x^2+8x}$

47) $\lim\limits_{x\to2}\dfrac{x+1}{x-2}$

48) $\lim\limits_{x\to3}\dfrac{x}{(x-3)^2}$

49) $\lim\limits_{x\to-1}\dfrac{2x}{(x+1)(x-2)}$

50) $\lim\limits_{x\to5}\dfrac{2x+1}{x^2-25}$

51) $\lim\limits_{x\to1}\dfrac{-(x+1)}{(x-1)^2}$

52) $\lim\limits_{x\to0}\dfrac{x}{\sqrt{x+9}-3}$

53) $\lim\limits_{x\to-2}\dfrac{2-\sqrt{6+x}}{x+2}$

54) $\lim\limits_{x\to3}\dfrac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9}$

55) $\lim\limits_{x\to4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$

56) $\lim\limits_{x\to9}\dfrac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}}$

57) $\lim\limits_{x\to1}\dfrac{2-\sqrt{x+3}}{\sqrt{x}-1}$

1) $\lim\limits_{n\to\infty}(n^2+2n+5) = [\infty + 2\cdot\infty + 5] = \underline{\underline{\infty}}$

2) $\lim\limits_{n\to\infty}(n^3-2n+5) = [\underbrace{\infty - 2\infty}_{\text{neurčitý výraz}} + 5] = \lim\limits_{n\to\infty} n^3\left(1-\frac{2n}{n^3}+\frac{5}{n^3}\right)$

$= \lim\limits_{n\to\infty} n^3\left(1-\frac{2}{n^2}+\frac{5}{n^3}\right) = [\infty\cdot(1-0+0)] = \underline{\underline{\infty}}$

3) $\lim\limits_{n\to\infty}\left(\frac{4}{n+1}-n\right) = \left[\frac{4}{\infty}-\infty\right] = [0-\infty] = \underline{\underline{-\infty}}$

4) $\lim\limits_{n\to\infty}\frac{2n^3+5n-1}{4n^3+2n-5} = \lim\limits_{n\to\infty}\frac{n^3\left(2n+\frac{5}{n^2}-\frac{1}{n^3}\right)}{n^3\left(4+\frac{2}{n^2}-\frac{5}{n^3}\right)} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$

(lze poznat výsledek již ze zadání, nebo můžeme počítat i L'hospitalem, protože $\frac{\infty}{\infty}$)

5) $\lim\limits_{n\to\infty}\frac{2n^2+5n}{n^4-2n+1} = \lim\limits_{n\to\infty}\frac{n^2\left(2+\frac{5}{n}\right)}{n^4\left(1-\frac{2}{n^3}+\frac{1}{n^4}\right)} = \lim\limits_{n\to\infty}\frac{2+\frac{5}{n}}{n^2\left(1-\frac{2}{n^3}+\frac{1}{n^4}\right)} = \left[\frac{2}{\infty\cdot 1}\right] = \underline{\underline{0}}$

(lze poznat ze zadání → větší mocnina ve jmenov.)

6) $\lim\limits_{n\to\infty}\frac{3n^3-2n+1}{-2n^2+n+5} = \lim\limits_{n\to\infty}\frac{n^3\left(3-\frac{2}{n^2}+\frac{1}{n^3}\right)}{n^2\left(-2+\frac{1}{n}+\frac{5}{n^2}\right)} = \lim\limits_{n\to\infty}\frac{n\left(3-\frac{2}{n^2}+\frac{1}{n^3}\right)}{-2+\frac{1}{n}+\frac{5}{n^2}} = \left[\frac{\infty\cdot 3}{-2}\right] = \underline{\underline{-\infty}}$

(lze poznat ze zadání → větší mocnina v čitateli)

7) $\lim\limits_{n\to\infty}\frac{-n^3-5n+1}{-2n+2} = \underline{\underline{+\infty}}$

8) $\lim\limits_{n\to\infty}\frac{5n-4}{2n+2} = \underline{\underline{\frac{5}{2}}}$

9) $\lim\limits_{n\to\infty}\frac{n^2+5n-1}{4n^5-2n+2} = \underline{\underline{0}}$

10) $\lim\limits_{n\to\infty}\frac{2^n+3^n}{4^n-2^n} = \lim\limits_{n\to\infty}\frac{3^n\left(\left(\frac{2}{3}\right)^n+1\right)}{4^n\left(1-\left(\frac{2}{4}\right)^n\right)} = \lim\limits_{n\to\infty}\left(\frac{3}{4}\right)^n\frac{\left(\frac{2}{3}\right)^n+1}{1-\left(\frac{1}{2}\right)^n} = \left[0\cdot\frac{1}{1}\right] = \underline{\underline{0}}$

11) $\lim\limits_{n\to\infty}\frac{3^n-2\cdot 5^n+1}{4^n+3\cdot 2^n} = \lim\limits_{n\to\infty}\frac{5^n\left(\left(\frac{3}{5}\right)^n-2+\left(\frac{1}{5}\right)^n\right)}{4^n\left(1+3\cdot\left(\frac{2}{4}\right)^n\right)} = \lim\limits_{n\to\infty}\left(\frac{5}{4}\right)^n\cdot\frac{\left(\frac{3}{5}\right)^n-2+\left(\frac{1}{5}\right)^n}{1+3\cdot\left(\frac{1}{2}\right)^n} = \left[\infty\cdot\frac{-2}{1}\right] = \underline{\underline{-\infty}}$

$2^{2n} = (2^2)^n = 4^n$

12) $\lim\limits_{n\to\infty}\frac{2^{2n}+3^{3n}}{3^{2n}-2^{3n}} = \lim\limits_{n\to\infty}\frac{4^n+27^n}{9^n-8^n} = \lim\limits_{n\to\infty}\frac{27^n\left(\left(\frac{4}{27}\right)^n+1\right)}{9^n\left(1-\left(\frac{8}{9}\right)^n\right)} = \lim\limits_{n\to\infty}\left(\frac{27}{9}\right)^n\frac{\left(\frac{4}{27}\right)^n+1}{1-\left(\frac{8}{9}\right)^n} = \left[\infty\cdot\frac{1}{1}\right] = \underline{\underline{\infty}}$

13) $\lim\limits_{n\to\infty} 2^n - 4^n + 5^n = \lim\limits_{n\to\infty} 5^n\left(\left(\frac{2}{5}\right)^n - \left(\frac{4}{5}\right)^n + 1\right) = [\infty \cdot 1] = \underline{\underline{\infty}}$

14) $\lim\limits_{n\to\infty} \frac{5\cdot 3^{2n} - 2^n}{3\cdot 2^n - 9^n} = \lim\limits_{n\to\infty} \frac{5\cdot 9^n - 2^n}{3\cdot 2^n - 9^n} = \lim\limits_{n\to\infty} \frac{\cancel{9^n}\left(5 - \left(\frac{2}{9}\right)^n\right)}{\cancel{9^n}\left(3\cdot\left(\frac{2}{9}\right)^n - 1\right)} = \underline{\underline{-5}}$

15) $\lim\limits_{n\to\infty} \frac{n!}{(n-1)!} = \lim\limits_{n\to\infty} \frac{n\cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = \underline{\underline{\infty}}$

16) $\lim\limits_{n\to\infty} \frac{(n+3)!}{(n+4)!} = \lim\limits_{n\to\infty} \frac{\cancel{(n+3)!}}{(n+4)\cdot\cancel{(n+3)!}} = \left[\frac{1}{\infty}\right] = \underline{\underline{0}}$

17) $\lim\limits_{n\to\infty} \frac{n! + (n-1)!}{(n+1)!} = \lim\limits_{n\to\infty} \frac{n\cdot(n-1)! + (n-1)!}{(n+1)(n)(n-1)!} =$

$\lim\limits_{n\to\infty} \frac{\cancel{(n-1)!}\,\cancel{(n+1)}}{\cancel{(n+1)}\cdot n\,\cancel{(n-1)!}} = \left[\frac{1}{\infty}\right] = \underline{\underline{0}}$

18) $\lim\limits_{n\to\infty} \frac{(n-2)! + (n-1)!}{(n+1)!} = \lim\limits_{n\to\infty} \frac{(n-2)! + (n-1)(n-2)!}{(n+1)\cdot n(n-1)(n-2)!} =$

$= \lim\limits_{n\to\infty} \frac{\cancel{(n-2)!}\,(1+n-1)}{(n+1)n(n-1)\cancel{(n-2)!}} = \lim\limits_{n\to\infty} \frac{\cancel{n}}{(n+1)\cdot\cancel{n}\cdot(n-1)} = \left[\frac{1}{\infty}\right] = \underline{\underline{0}}$

19) $\lim\limits_{n\to\infty} \frac{(n+2)! + n!}{(n+1)!} = \lim\limits_{n\to\infty} \frac{(n+2)(n+1)n! + n!}{(n+1)\cdot n!} =$

$= \lim\limits_{n\to\infty} \frac{\cancel{n!}\,(n^2+2n+n+2+1)}{(n+1)\cdot\cancel{n!}} = \lim\limits_{n\to\infty} \frac{n^2+3n+3}{n+1} = \underline{\underline{+\infty}}$

(větší mocnina nahoře)

20) $\lim\limits_{n\to\infty} \frac{n! - (n-1)!}{(n-1)!} = \lim\limits_{n\to\infty} \frac{n\cdot(n-1)! - (n-1)!}{(n-1)!} =$

$= \lim\limits_{n\to\infty} \frac{\cancel{(n-1)!}\cdot(n-1)}{\cancel{(n-1)!}} = \underline{\underline{\infty}}$

21) $\lim\limits_{n\to\infty} \frac{n! + (n-1)!}{(n+1)! - n!} = \lim\limits_{n\to\infty} \frac{n\cdot(n-1)! + (n-1)!}{(n+1)\cdot n(n-1)! - n\cdot(n-1)!} =$

$= \lim\limits_{n\to\infty} \frac{\cancel{(n-1)!}\,(n+1)}{\cancel{(n-1)!}\,[n^2+n-n]} = \lim\limits_{n\to\infty} \frac{n+1}{n^2} = \underline{\underline{0}}$ (větší mocnina ve jmenovateli)

22) $\lim\limits_{n\to\infty}\left(1+\frac{4}{n}\right)^2 = \underline{1}$ $\quad\left[\left(1+\frac{4}{\infty}\right)^2 = (1+0)^2 = 1\right]$

23) $\lim\limits_{n\to\infty}\left(1+\frac{3}{n}\right)^n = \underline{\underline{e^3}}$! vzorec

$$\boxed{\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k}$$

24) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n}\right)^n = \lim\limits_{n\to\infty}\left(1-\frac{4}{n}\right)^n = \underline{\underline{e^{-4}}}$

25) $\lim\limits_{n\to\infty}\left(\frac{n}{n-2}\right)^n = \lim\limits_{n\to\infty}\frac{1}{\left(\frac{n-2}{n}\right)^n} = \lim\limits_{n\to\infty}\frac{1}{\left(1-\frac{2}{n}\right)^n} = \frac{1}{e^{-2}} = \underline{\underline{e^2}}$

26) $\lim\limits_{n\to\infty}\left(\frac{2n}{2n+4}\right)^n = \lim\limits_{n\to\infty}\frac{1}{\left(\frac{2n+4}{2n}\right)^n} = \lim\limits_{n\to\infty}\frac{1}{\left(1+\frac{2}{n}\right)^n} = \underline{\underline{\frac{1}{e^2}}}$

27) $\lim\limits_{n\to\infty}\left(\frac{n-3}{n+1}\right)^n = \lim\limits_{n\to\infty}\left(\frac{\cancel{n}\left(1-\frac{3}{n}\right)}{\cancel{n}\left(1+\frac{1}{n}\right)}\right)^n = \lim\limits_{n\to\infty}\frac{\left(1-\frac{3}{n}\right)^n}{\left(1+\frac{1}{n}\right)^n} = \frac{e^{-3}}{e^1} = \underline{\underline{\frac{1}{e^4}}}$

28) $\lim\limits_{n\to\infty}\left(\frac{2n+1}{n-2}\right)^n = \lim\limits_{n\to\infty}\left(\frac{2\cancel{n}\left(1+\frac{1}{2n}\right)}{\cancel{n}\left(1-\frac{2}{n}\right)}\right)^n = \lim\limits_{n\to\infty} 2^n\cdot\frac{\left(1+\frac{1/2}{n}\right)^n}{\left(1-\frac{2}{n}\right)^n} = \left[\infty\cdot\frac{e^{1/2}}{e^{-2}}\right] = \underline{\underline{+\infty}}$

29) $\lim\limits_{n\to\infty}\left(\frac{3n-2}{4n+12}\right)^n = \lim\limits_{n\to\infty}\left(\frac{3n\left(1-\frac{2}{3n}\right)}{4n\left(1+\frac{12}{4n}\right)}\right)^n = \lim\limits_{n\to\infty}\left(\frac{3}{4}\right)^n\cdot\frac{\left(1-\frac{2/3}{n}\right)^n}{\left(1+\frac{3}{n}\right)^n} = \left[0\cdot\frac{e^{-2/3}}{e^3}\right] = \underline{\underline{0}}$

30) $\lim\limits_{n\to\infty}\left(\frac{4n-1}{3n+1}\right)^n = \lim\limits_{n\to\infty}\left(\frac{4n\left(1-\frac{1}{4n}\right)}{3n\left(1+\frac{1}{3n}\right)}\right)^n = \lim\limits_{n\to\infty}\left(\frac{4}{3}\right)^n\frac{\left(1-\frac{1/4}{n}\right)^n}{\left(1+\frac{1/3}{n}\right)^n} = \left[\infty\cdot\frac{e^{-1/4}}{e^{1/3}}\right] = \underline{\underline{\infty}}$

31) $\lim\limits_{n\to\infty}\left(\frac{2n+3}{2n-2}\right)^n = \lim\limits_{n\to\infty}\left(\frac{2n\left(1+\frac{3}{2n}\right)}{2n\left(1-\frac{2}{2n}\right)}\right)^n = \lim\limits_{n\to\infty}\frac{\left(1+\frac{3/2}{n}\right)^n}{\left(1-\frac{1}{n}\right)^n} = \frac{e^{\frac{3}{2}}}{e^{-1}} = e^{\frac{3}{2}}\cdot e^1 = e^{\frac{5}{2}} = \underline{\underline{\sqrt{e^5}}}$

32) $\lim\limits_{n\to\infty}\sqrt{\frac{n-3}{n^2+1}} = \lim\limits_{n\to\infty}\sqrt{\frac{n\left(1-\frac{3}{n}\right)}{n^2\left(1+\frac{1}{n^2}\right)}} = \lim\limits_{n\to\infty}\sqrt{\frac{1-\frac{3}{n}}{n\left(1+\frac{1}{n^2}\right)}} = \left[\sqrt{\frac{1}{\infty\cdot 1}} = \sqrt{0}\right] = \underline{\underline{0}}$

33) $\lim\limits_{x\to\infty}\frac{\sqrt{x^4-2x}}{\sqrt{x^2+2x-1}} = \lim\limits_{x\to\infty}\frac{\sqrt{x^4\left(1-\frac{2}{x^3}\right)}}{\sqrt{x^2\left(1+\frac{2}{x}-\frac{1}{x^2}\right)}} = \lim\limits_{x\to\infty}\frac{x^{\cancel{2}}\sqrt{1-\frac{2}{x^3}}}{\cancel{x}\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}} = \left[\frac{\infty\cdot 1}{1}\right] = \underline{\underline{\infty}}$

34) $\lim\limits_{n\to\infty} \sqrt[n]{2n^4} = \lim\limits_{n\to\infty} \sqrt[n]{2}\cdot\sqrt[n]{n^4} = \lim\limits_{n\to\infty} \underset{\downarrow\,1}{\sqrt[n]{2}}\cdot\underset{\downarrow\,1}{\left(\sqrt[n]{n}\right)}^4 = \underline{\underline{1}}$

VZOREC!

$\boxed{\lim\limits_{n\to\infty} \sqrt[n]{n} = 1}$ $\boxed{\lim\limits_{n\to\infty} \sqrt[n]{a} = 1}$

a je kladné číslo

35) $\lim\limits_{n\to\infty} \sqrt[n]{n^3-2n+1} = \lim\limits_{n\to\infty} \sqrt[n]{n^3\left(1-\frac{2}{n^2}+\frac{1}{n^3}\right)} =$

$= \lim\limits_{n\to\infty} \sqrt[n]{n^3}\cdot\sqrt[n]{1-\underset{\downarrow\,0}{\frac{2}{n^2}}+\underset{\downarrow\,0}{\frac{1}{n^3}}} = \left[1^3\cdot 1\right] = \underline{\underline{1}}$

36) $\lim\limits_{n\to\infty} \sqrt[n]{5n^3} = \lim\limits_{n\to\infty} \sqrt[n]{5}\cdot\left(\sqrt[n]{n}\right)^3 = \underline{\underline{1}}$

37) $\lim\limits_{n\to\infty} \sqrt[n]{2n+4} = \lim\limits_{n\to\infty} \sqrt[n]{n\left(2+\frac{4}{n}\right)} = \lim\limits_{n\to\infty} \sqrt[n]{n}\cdot\sqrt[n]{2+\underset{\downarrow\,0}{\frac{4}{n}}} = \underline{\underline{1}}$

38) $\lim\limits_{n\to\infty} \sqrt{n^2+2n}+n = [\infty+\infty] = \underline{\underline{\infty}}$

39) $\lim\limits_{n\to\infty} \sqrt{n^2+2n}-n = [\infty-\infty] =$ musíme rozšíriť

$= \lim\limits_{n\to\infty} \left(\sqrt{n^2+2n}-n\right)\cdot\frac{\sqrt{n^2+2n}+n}{\sqrt{n^2+2n}+n} = \lim\limits_{n\to\infty} \frac{n^2+2n-n^2}{\sqrt{n^2+2n}+n} =$

$= \lim\limits_{n\to\infty} \frac{2n}{\sqrt{n^2\left(1+\frac{2}{n}\right)}+n} = \lim\limits_{n\to\infty} \frac{2n}{n\sqrt{1+\frac{2}{n}}+n} = \lim\limits_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{2}{n}}+1\right)} = \left[\frac{2}{\sqrt{1}+1}\right] = \underline{\underline{1}}$

40) $\lim\limits_{n\to\infty} \sqrt{n^2+1}-n = [\infty-\infty] = \lim\limits_{n\to\infty} \left(\sqrt{n^2+1}-n\right)\cdot\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n} =$

$= \lim\limits_{n\to\infty} \frac{n^2+1-n^2}{\sqrt{n^2+1}+n} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+1}+n} = \left[\frac{1}{\infty}\right] = \underline{\underline{0}}$

41) $\lim\limits_{n\to\infty} \sqrt{n+3}-\sqrt{n-1} = [\infty-\infty] = \lim\limits_{n\to\infty} \left(\sqrt{n+3}-\sqrt{n-1}\right)\cdot\frac{\sqrt{n+3}+\sqrt{n-1}}{\sqrt{n+3}+\sqrt{n-1}}$

$= \lim\limits_{n\to\infty} \frac{n+3-(n-1)}{\sqrt{n+3}+\sqrt{n-1}} = \lim\limits_{n\to\infty} \frac{4}{\sqrt{n+3}+\sqrt{n-1}} = \left[\frac{4}{\infty}\right] = \underline{\underline{0}}$

42) $\lim\limits_{x\to 3} \dfrac{x^2+3}{x+3} = \dfrac{9+3}{3+3} = \dfrac{12}{6} = \underline{\underline{2}}$

43) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \underline{\underline{6}}$

↳ lze také L'hospitalem ⟹

$\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x}{1} = 2\cdot 3 = \underline{\underline{6}}$

44) $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{81-x^4} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{(x-3)^2}{(9-x^2)(9+x^2)} = \lim\limits_{x\to 3} \dfrac{(x-3)(x-3)}{(3-x)(3+x)(9+x^2)} =$

$= \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-3)}{-\cancel{(x-3)}(3+x)(9+x^2)} = \dfrac{0}{-6\cdot 18} = \underline{\underline{0}}$

nebo L'hospitalem ⟹

$\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x-6}{-4x^3} = \dfrac{2\cdot 3-6}{-4\cdot 27} = \underline{\underline{0}}$

45) $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x^2-8x+15} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}(x-5)} = \dfrac{8}{-2} = \underline{\underline{-4}}$

↳ $\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x+2}{2x-8} = \dfrac{6+2}{6-8} = \dfrac{8}{-2} = \underline{\underline{-4}}$

46) $\lim\limits_{x\to -2} \dfrac{x^2-x-6}{x^3+6x^2+8x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x-3)}{x\cdot\cancel{(x+2)}(x+4)} = \dfrac{-5}{-2\cdot 2} = \underline{\underline{\dfrac{5}{4}}}$

↳ $\overset{LH}{=} \lim\limits_{x\to -2} \dfrac{2x-1}{3x^2+12x+8} = \dfrac{-5}{3\cdot 4-24+8} = \underline{\underline{\dfrac{5}{4}}}$

47) $\lim\limits_{x\to 2} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0}\right]$! typ $\dfrac{\text{číslo}}{0}$ ⟹ jednostranné limity

$\lim\limits_{x\to 2+} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0+}\right] = +\infty$ ← malé kladné číslo

$\lim\limits_{x\to 2-} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0-}\right] = -\infty$

↑ malé záporné č.

liší se ⟹ původní limita $\lim\limits_{x\to 2} \dfrac{x+1}{x-2}$ neexistuje

48) $\lim\limits_{x\to 3} \frac{x}{(x-3)^2} = \left[\frac{3}{0}\right]$ → jednostranné limity

$\lim\limits_{x\to 3+} \frac{x}{(x-3)^2} = \left[\frac{3}{0+}\right] = +\infty$

$\lim\limits_{x\to 3-} \frac{x}{(x-3)^2} = \left[\frac{3}{0+}\right] = +\infty$

} i pôvodná limita $\lim\limits_{x\to 3} \frac{x}{(x-3)^2} = \underline{\underline{+\infty}}$

49) $\lim\limits_{x\to -1} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0}\right]$ ⇒ jednostr. lim.

$\lim\limits_{x\to -1+} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0+\cdot(-3)}\right] = +\infty$

$\lim\limits_{x\to -1-} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0-\cdot(-3)}\right] = -\infty$

} $\lim\limits_{x\to -1} \frac{2x}{(x+1)(x-2)}$ neexistuje

50) $\lim\limits_{x\to 5} \frac{2x+1}{x^2-25} = \left[\frac{11}{0}\right]$ ⇒ jednostr. lim.

$\lim\limits_{x\to 5+} \frac{2x+1}{x^2-25} = \left[\frac{11}{0+}\right] = +\infty$

$\lim\limits_{x\to 5-} \frac{2x+1}{x^2-25} = \left[\frac{11}{0-}\right] = -\infty$

} $\lim\limits_{x\to 5} \frac{2x+1}{x^2-25}$ = neex.

51) $\lim\limits_{x\to 1} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0}\right]$ ⇒ jednostr. lim.

$\lim\limits_{x\to 1+} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0+}\right] = -\infty$

$\lim\limits_{x\to 1-} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0+}\right] = -\infty$

} $\lim\limits_{x\to 1} \frac{-(x+1)}{(x-1)^2} = \underline{\underline{-\infty}}$

52) $\lim\limits_{x\to 0} \frac{x}{\sqrt{x+9}-3} = \left[\frac{0}{0}\right]$ = ! rozširujeme

$$\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x+9-9} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x} = \underline{\underline{6}}$$

53) $\lim\limits_{x\to -2} \dfrac{2-\sqrt{6+x}}{x+2} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{2-\sqrt{6+x}}{x+2} \cdot \dfrac{2+\sqrt{6+x}}{2+\sqrt{6+x}}$

$= \lim\limits_{x\to -2} \dfrac{4-(6+x)}{(x+2)(2+\sqrt{6+x})} = \lim\limits_{x\to -2} \dfrac{\cancel{-2-x}}{-(\cancel{-x-2})(\sqrt{6+x}+2)} = \dfrac{1}{-(2+2)} = -\dfrac{1}{4}$

54) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9} \cdot \dfrac{\sqrt{x+6}+3\sqrt{x-2}}{\sqrt{x+6}+3\sqrt{x-2}}$

$= \lim\limits_{x\to 3} \dfrac{x+6-9(x-2)}{(x^2-9)(\sqrt{x+6}+3\sqrt{x-2})} = \lim\limits_{x\to 3} \dfrac{-8x+24}{(x^2-9)(\sqrt{x+6}+3\sqrt{x-2})} =$

$= \lim\limits_{x\to 3} \dfrac{-8(\cancel{x-3})}{(\cancel{x-3})(x+3)(\sqrt{x+6}+3\sqrt{x-2})} = \dfrac{-8}{6\cdot(3+3\cdot 1)} = \dfrac{-8}{36} = \dfrac{-2}{9}$

55) $\lim\limits_{x\to 4} \dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 4} \dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \lim\limits_{x\to 4}$

$\dfrac{(\sqrt{1+2x}-3)(\sqrt{x}+2)}{x-4} \cdot \dfrac{\sqrt{1+2x}+3}{\sqrt{1+2x}+3} = \lim\limits_{x\to 4} \dfrac{(1+2x-9)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)}$

$= \lim\limits_{x\to 4} \dfrac{(-8+2x)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)} = \lim\limits_{x\to 4} \dfrac{2(\cancel{x-4})(\sqrt{x}+2)}{(\cancel{x-4})(\sqrt{1+2x}+3)} = \dfrac{2\cdot 4}{6} = \dfrac{4}{3}$

56) $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}} \cdot \dfrac{\sqrt{2x}+3\sqrt{2}}{\sqrt{2x}+3\sqrt{2}} =$

$= \lim\limits_{x\to 9} \dfrac{(\sqrt{x}-3)(\sqrt{2x}+3\sqrt{2})}{2x-9\cdot 2} = \lim\limits_{x\to 9} \dfrac{(\sqrt{x}-3)\cdot(\sqrt{2x}+3\sqrt{2})}{2x-18} \cdot \dfrac{\sqrt{x}+3}{\sqrt{x}+3} =$

$= \lim\limits_{x\to 9} \dfrac{(\sqrt{2x}+3\sqrt{2})(\cancel{x-9})}{2(\cancel{x-9})(\sqrt{x}+3)} = \dfrac{\sqrt{18}+3\sqrt{2}}{2\cdot 6} = \dfrac{3\sqrt{2}+3\sqrt{2}}{12} = \dfrac{\sqrt{2}}{2}$

57) $\lim\limits_{x\to 1} \dfrac{2-\sqrt{x+3}}{\sqrt{x}-1} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{2-\sqrt{x+3}}{\sqrt{x}-1} \cdot \dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \lim\limits_{x\to 1} \dfrac{(2-\sqrt{x+3})(\sqrt{x}+1)}{x-1}$

$= \lim\limits_{x\to 1} \dfrac{(2-\sqrt{x+3})(\sqrt{x}+1)}{x-1} \cdot \dfrac{2+\sqrt{x+3}}{2+\sqrt{x+3}} = \lim\limits_{x\to 1} \dfrac{(4-(x+3))(\sqrt{x}+1)}{(x-1)(2+\sqrt{x+3})} =$

$= \lim\limits_{x\to 1} \dfrac{(\cancel{1-x})(\sqrt{x}+1)}{-(\cancel{-x+1})(2+\sqrt{x+3})} = \dfrac{2}{-(2+2)} = -\dfrac{1}{2}$