LIMITY

1) $\lim\limits_{n\to\infty}\left(n^2+2n+5\right)$

2) $\lim\limits_{n\to\infty}\left(n^3-2n+5\right)$

3) $\lim\limits_{n\to\infty}\left(\frac{4}{n+1}-n\right)$

4) $\lim\limits_{n\to\infty}\frac{2n^3+5n-1}{4n^3+2n-5}$

5) $\lim\limits_{n\to\infty}\frac{2n^2+5n}{n^4-2n+1}$

6) $\lim\limits_{n\to\infty}\frac{3n^3-2n+1}{-2n^2+n+5}$

7) $\lim\limits_{n\to\infty}\frac{-n^3-5n+1}{-2n+2}$

8) $\lim\limits_{n\to\infty}\frac{5n-4}{2n+2}$

9) $\lim\limits_{n\to\infty}\frac{n^2+5n-1}{4n^5-2n+2}$

10) $\lim\limits_{n\to\infty}\frac{2^n+3^n}{4^n-2^n}$

11) $\lim\limits_{n\to\infty}\frac{3^n-2\cdot 5^n+1}{4^n+3\cdot 2^n}$

12) $\lim\limits_{n\to\infty}\frac{2^{2n}+3^{3n}}{3^{2n}-2^{3n}}$

13) $\lim\limits_{n\to\infty}\left(2^n-4^n+5^n\right)$

14) $\lim\limits_{n\to\infty}\frac{5\cdot 3^{2n}-2^n}{3\cdot 2^n-9^n}$

15) $\lim\limits_{n\to\infty}\sqrt{\frac{n-3}{n^2+1}}$

16) $\lim\limits_{x\to\infty}\frac{\sqrt{x^4-2x}}{\sqrt{x^2+2x-1}}$

17) $\lim\limits_{n\to\infty}\sqrt{n^2+2n}+n$

18) $\lim\limits_{n\to\infty}\sqrt{n^2+2n}-n$

19) $\lim\limits_{n\to\infty}\sqrt{n^2+1}-n$

20) $\lim\limits_{n\to\infty}\sqrt{n+3}-\sqrt{n-1}$

21) $\lim\limits_{x\to 3}\frac{x^2+3}{x+3}$

22) $\lim\limits_{x\to 3}\frac{x^2-9}{x-3}$

23) $\lim\limits_{x\to 3}\frac{x^2-6x+9}{81-x^4}$

24) $\lim\limits_{x\to 3}\frac{x^2+2x-15}{x^2-8x+15}$

25) $\lim\limits_{x\to -2}\frac{x^2-x-6}{x^3+6x^2+8x}$

26) $\lim\limits_{x\to 2}\frac{x+1}{x-2}$

27) $\lim\limits_{x\to 3}\frac{x}{(x-3)^2}$

28) $\lim\limits_{x\to -1}\frac{2x}{(x+1)(x-2)}$

29) $\lim\limits_{x\to 5}\frac{2x+1}{x^2-25}$

30) $\lim\limits_{x\to 1}\frac{-(x+1)}{(x-1)^2}$

31) $\lim\limits_{x\to 0}\frac{x}{\sqrt{x+9}-3}$

32) $\lim\limits_{x\to -2}\frac{2-\sqrt{6+x}}{x+2}$

33) $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9}$

34) $\lim\limits_{x\to 4}\frac{\sqrt{1+2x}-3}{\sqrt{x}-2}$

35) $\lim\limits_{x\to 9}\frac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}}$

36) $\lim\limits_{x\to 1}\frac{2-\sqrt{x+3}}{\sqrt{x}-1}$

Limity posloupností a funkcí:

1) $\lim\limits_{n\to\infty} (n^2+2n+5) = [\infty + 2\cdot\infty + 5] = \underline{\underline{\infty}}$

2) $\lim\limits_{n\to\infty} (n^3-2n+5) = [\underbrace{\infty - 2\infty}_{\text{neurčitý výraz}} + 5] = \lim\limits_{n\to\infty} n^3\left(1-\frac{2n}{n^3}+\frac{5}{n^3}\right)$

$= \lim\limits_{n\to\infty} n^3\left(1-\frac{2}{n^2}+\frac{5}{n^3}\right) = [\infty\cdot(1-0+0)] = \underline{\underline{\infty}}$

3) $\lim\limits_{n\to\infty} \left(\frac{4}{n+1}-n\right) = \left[\frac{4}{\infty}-\infty\right] = [0-\infty] = \underline{\underline{-\infty}}$

4) $\lim\limits_{n\to\infty} \frac{2n^3+5n-1}{4n^3+2n-5} = \lim\limits_{n\to\infty} \frac{\cancel{n^3}\left(2+\frac{5}{n^2}-\frac{1}{n^3}\right)}{\cancel{n^3}\left(4+\frac{2}{n^2}-\frac{5}{n^3}\right)} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$

(lze poznat výsledek již ze zadání, nebo můžeme počítat i L'hospitalem, protože $\frac{\infty}{\infty}$)

5) $\lim\limits_{n\to\infty} \frac{2n^2+5n}{n^4-2n+1} = \lim\limits_{n\to\infty} \frac{n^2\left(2+\frac{5}{n}\right)}{n^4\left(1-\frac{2}{n^3}+\frac{1}{n^4}\right)} = \lim\limits_{n\to\infty} \frac{2+\frac{5}{n}}{n^2\left(1-\frac{2}{n^3}+\frac{1}{n^4}\right)} = \left[\frac{2}{\infty\cdot 1}\right] = \underline{\underline{0}}$

(lze poznat ze zadání → větší mocnina ve jmenov.)

6) $\lim\limits_{n\to\infty} \frac{3n^3-2n+1}{-2n^2+n+5} = \lim\limits_{n\to\infty} \frac{n^3\left(3-\frac{2}{n^2}+\frac{1}{n^3}\right)}{n^2\left(-2+\frac{1}{n}+\frac{5}{n^2}\right)} = \lim\limits_{n\to\infty} \frac{n\left(3-\frac{2}{n^2}+\frac{1}{n^3}\right)}{-2+\frac{1}{n}+\frac{5}{n^2}} = \left[\frac{\infty\cdot 3}{-2}\right] = \underline{\underline{-\infty}}$

(lze poznat ze zadání → větší mocnina v čitateli)

7) $\lim\limits_{n\to\infty} \frac{-n^3-5n+1}{-2n+2} = \underline{\underline{+\infty}}$

8) $\lim\limits_{n\to\infty} \frac{5n-4}{2n+2} = \underline{\underline{\frac{5}{2}}}$

9) $\lim\limits_{n\to\infty} \frac{n^2+5n-1}{4n^5-2n+2} = \underline{\underline{0}}$

10) $\lim\limits_{n\to\infty} \frac{2^n+3^n}{4^n-2^n} = \lim\limits_{n\to\infty} \frac{3^n\left(\left(\frac{2}{3}\right)^n+1\right)}{4^n\left(1-\left(\frac{2}{4}\right)^n\right)} = \lim\limits_{n\to\infty} \left(\frac{3}{4}\right)^n \frac{\left(\frac{2}{3}\right)^n+1}{1-\left(\frac{1}{2}\right)^n} = \left[0\cdot\frac{1}{1}\right] = \underline{\underline{0}}$

11) $\lim\limits_{n\to\infty} \frac{3^n-2\cdot 5^n+1}{4^n+3\cdot 2^n} = \lim\limits_{n\to\infty} \frac{5^n\left(\left(\frac{3}{5}\right)^n-2+\left(\frac{1}{5}\right)^n\right)}{4^n\left(1+3\cdot\left(\frac{2}{4}\right)^n\right)} = \lim\limits_{n\to\infty} \left(\frac{5}{4}\right)^n \cdot \frac{\left(\frac{3}{5}\right)^n-2+\left(\frac{1}{5}\right)^n}{1+3\cdot\left(\frac{1}{2}\right)^n} = \left[\infty\cdot\frac{-2}{1}\right] = \underline{\underline{-\infty}}$

$2^{2n} = (2^2)^n = 4^n$

12) $\lim\limits_{n\to\infty} \frac{2^{2n}+3^{3n}}{3^{2n}-2^{3n}} = \lim\limits_{n\to\infty} \frac{4^n+27^n}{9^n-8^n} = \lim\limits_{n\to\infty} \frac{27^n\left(\left(\frac{4}{27}\right)^n+1\right)}{9^n\left(1-\left(\frac{8}{9}\right)^n\right)} = \lim\limits_{n\to\infty} \left(\frac{27}{9}\right)^n \frac{\left(\frac{4}{27}\right)^n+1}{1-\left(\frac{8}{9}\right)^n} = \left[\infty\cdot\frac{1}{1}\right] = \underline{\underline{\infty}}$

13) $\lim\limits_{n\to\infty} 2^n - 4^n + 5^n = \lim\limits_{n\to\infty} 5^n\left(\left(\frac{2}{5}\right)^n - \left(\frac{4}{5}\right)^n + 1\right) = [\infty\cdot 1] = \underline{\underline{\infty}}$

14) $\lim\limits_{n\to\infty} \frac{5\cdot 3^{2n} - 2^n}{3\cdot 2^n - 9^n} = \lim\limits_{n\to\infty} \frac{5\cdot 9^n - 2^n}{3\cdot 2^n - 9^n} = \lim\limits_{n\to\infty} \frac{\cancel{9^n}\left(5 - \left(\frac{2}{9}\right)^n\right)}{\cancel{9^n}\left(3\cdot\left(\frac{2}{9}\right)^n - 1\right)} = \underline{\underline{-5}}$

$= e^{\frac{5}{2}} = \underline{\underline{\sqrt{e^5}}}$

32) $\lim\limits_{n\to\infty} \sqrt{\frac{n-3}{n^2+1}} = \lim\limits_{n\to\infty} \sqrt{\frac{n\left(1-\frac{3}{n}\right)}{n^2\left(1+\frac{1}{n^2}\right)}} = \lim\limits_{n\to\infty} \sqrt{\frac{1-\frac{3}{n}}{n\left(1+\frac{1}{n^2}\right)}} = \left[\sqrt{\frac{1}{\infty\cdot 1}} = \sqrt{0}\right] = \underline{\underline{0}}$

33) $\lim\limits_{x\to\infty} \frac{\sqrt{x^4-2x}}{\sqrt{x^2+2x-1}} = \lim\limits_{x\to\infty} \frac{\sqrt{x^4\left(1-\frac{2}{x^3}\right)}}{\sqrt{x^2\left(1+\frac{2}{x}-\frac{1}{x^2}\right)}} = \lim\limits_{x\to\infty} \frac{x^{\cancel{2}}\sqrt{1-\frac{2}{x^3}}}{\cancel{x}\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}} = \left[\frac{\infty\cdot 1}{1}\right] = \underline{\underline{\infty}}$

38) $\lim\limits_{n \to \infty} \sqrt{n^2+2n} + n = [\infty + \infty] = \underline{\underline{\infty}}$

39) $\lim\limits_{n \to \infty} \sqrt{n^2+2n} - n = [\infty - \infty] =$ musíme rozšířit

$= \lim\limits_{n \to \infty} \left(\sqrt{n^2+2n} - n\right) \cdot \dfrac{\sqrt{n^2+2n} + n}{\sqrt{n^2+2n} + n} = \lim\limits_{n \to \infty} \dfrac{n^2 + 2n - n^2}{\sqrt{n^2+2n} + n} =$

$= \lim\limits_{n \to \infty} \dfrac{2n}{\sqrt{n^2\left(1+\frac{2}{n}\right)} + n} = \lim\limits_{n \to \infty} \dfrac{2n}{n\sqrt{1+\frac{2}{n}} + n} = \lim\limits_{n \to \infty} \dfrac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{2}{n}} + 1\right)} = \left[\dfrac{2}{\sqrt{1}+1}\right] = \underline{\underline{1}}$

40) $\lim\limits_{n \to \infty} \sqrt{n^2+1} - n = [\infty - \infty] = \lim\limits_{n \to \infty} \left(\sqrt{n^2+1} - n\right) \cdot \dfrac{\sqrt{n^2+1} + n}{\sqrt{n^2+1} + n} =$

$= \lim\limits_{n \to \infty} \dfrac{n^2 + 1 - n^2}{\sqrt{n^2+1} + n} = \lim\limits_{n \to \infty} \dfrac{1}{\sqrt{n^2+1} + n} = \left[\dfrac{1}{\infty}\right] = \underline{\underline{0}}$

41) $\lim\limits_{n \to \infty} \sqrt{n+3} - \sqrt{n-1} = [\infty - \infty] = \lim\limits_{n \to \infty} \left(\sqrt{n+3} - \sqrt{n-1}\right) \cdot \dfrac{\sqrt{n+3} + \sqrt{n-1}}{\sqrt{n+3} + \sqrt{n-1}}$

$= \lim\limits_{n \to \infty} \dfrac{n + 3 - (n-1)}{\sqrt{n+3} + \sqrt{n-1}} = \lim\limits_{n \to \infty} \dfrac{4}{\sqrt{n+3} + \sqrt{n-1}} = \left[\dfrac{4}{\infty}\right] = \underline{\underline{0}}$

42) $\lim\limits_{x\to 3} \dfrac{x^2+3}{x+3} = \dfrac{9+3}{3+3} = \dfrac{12}{6} = \underline{\underline{2}}$

43) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \underline{\underline{6}}$

↳ lze také L'hospitalem ⇒

$\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x}{1} = 2\cdot 3 = \underline{\underline{6}}$

44) $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{81-x^4} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{(x-3)^2}{(9-x^2)(9+x^2)} = \lim\limits_{x\to 3} \dfrac{(x-3)(x-3)}{(3-x)(3+x)(9+x^2)} =$

$= \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-3)}{-\cancel{(x-3)}(3+x)(9+x^2)} = \dfrac{0}{-6\cdot 18} = \underline{\underline{0}}$

nebo L'hospitalem ⇒

$\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x-6}{-4x^3} = \dfrac{2\cdot 3-6}{-4\cdot 27} = \underline{\underline{0}}$

45) $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x^2-8x+15} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}(x-5)} = \dfrac{8}{-2} = \underline{\underline{-4}}$

↳ $\overset{LH}{=} \lim\limits_{x\to 3} \dfrac{2x+2}{2x-8} = \dfrac{6+2}{6-8} = \dfrac{8}{-2} = \underline{\underline{-4}}$

46) $\lim\limits_{x\to -2} \dfrac{x^2-x-6}{x^3+6x^2+8x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x-3)}{x\cdot\cancel{(x+2)}(x+4)} = \dfrac{-5}{-2\cdot 2} = \underline{\underline{\dfrac{5}{4}}}$

↳ $\overset{LH}{=} \lim\limits_{x\to -2} \dfrac{2x-1}{3x^2+12x+8} = \dfrac{-5}{3\cdot 4-24+8} = \underline{\underline{\dfrac{5}{4}}}$

47) $\lim\limits_{x\to 2} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0}\right]$! typ $\dfrac{\text{číslo}}{0}$ ⇒ jednostranné limity

$\lim\limits_{x\to 2+} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0+}\right] = +\infty$ ← malé kladné číslo

$\lim\limits_{x\to 2-} \dfrac{x+1}{x-2} = \left[\dfrac{3}{0-}\right] = -\infty$

↑ malé záporné č.

liší se ⇒ původní limita $\lim\limits_{x\to 2} \dfrac{x+1}{x-2}$ neexistuje

48) $\lim_{x\to 3} \frac{x}{(x-3)^2} = \left[\frac{3}{0}\right] \to$ jednostranné limity

$\lim_{x\to 3+} \frac{x}{(x-3)^2} = \left[\frac{3}{0+}\right] = +\infty$

$\lim_{x\to 3-} \frac{x}{(x-3)^2} = \left[\frac{3}{0+}\right] = +\infty$

$\Big\}$ i původní limita $\lim_{x\to 3} \frac{x}{(x-3)^2} = \underline{\underline{+\infty}}$

49) $\lim_{x\to -1} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0}\right] \Rightarrow$ jednostr. lim.

$\lim_{x\to -1+} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0+\cdot(-3)}\right] = +\infty$

$\lim_{x\to -1-} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0-\cdot(-3)}\right] = -\infty$

$\Big\}$ $\lim_{x\to -1} \frac{2x}{(x+1)(x-2)}$ neexistuje

50) $\lim_{x\to 5} \frac{2x+1}{x^2-25} = \left[\frac{11}{0}\right] \Rightarrow$ jednostr. lim.

$\lim_{x\to 5+} \frac{2x+1}{x^2-25} = \left[\frac{11}{0+}\right] = +\infty$

$\lim_{x\to 5-} \frac{2x+1}{x^2-25} = \left[\frac{11}{0-}\right] = -\infty$

$\Big\}$ $\lim_{x\to 5} \frac{2x+1}{x^2-25}$ = neex.

51) $\lim_{x\to 1} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0}\right] \Rightarrow$ jednostr. lim.

$\lim_{x\to 1+} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0+}\right] = -\infty$

$\lim_{x\to 1-} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0+}\right] = -\infty$

$\Big\}$ $\lim_{x\to 1} \frac{-(x+1)}{(x-1)^2} = \underline{\underline{-\infty}}$

52) $\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3} = \left[\frac{0}{0}\right] = !$ rozšiřujeme

$\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x+9-9} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x} = \underline{\underline{6}}$

53) $$\lim_{x\to -2} \frac{2-\sqrt{6+x}}{x+2} = \left[\frac{0}{0}\right] = \lim_{x\to -2} \frac{2-\sqrt{6+x}}{x+2} \cdot \frac{2+\sqrt{6+x}}{2+\sqrt{6+x}}$$

$$= \lim_{x\to -2} \frac{4-(6+x)}{(x+2)(2+\sqrt{6+x})} = \lim_{x\to -2} \frac{\cancel{-2-x}}{-\cancel{(-x-2)}(\sqrt{6+x}+2)} = \frac{1}{-(2+2)} = -\frac{1}{4}$$

54) $$\lim_{x\to 3} \frac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9} = \left[\frac{0}{0}\right] = \lim_{x\to 3} \frac{\sqrt{x+6}-3\sqrt{x-2}}{x^2-9} \cdot \frac{\sqrt{x+6}+3\sqrt{x-2}}{\sqrt{x+6}+3\sqrt{x-2}}$$

$$= \lim_{x\to 3} \frac{x+6-9(x-2)}{(x^2-9)(\sqrt{x+6}+3\sqrt{x-2})} = \lim_{x\to 3} \frac{-8x+24}{(x^2-9)(\sqrt{x+6}+3\sqrt{x-2})} =$$

$$= \lim_{x\to 3} \frac{-8\cancel{(x-3)}}{\cancel{(x-3)}(x+3)(\sqrt{x+6}+3\sqrt{x-2})} = \frac{-8}{6\cdot(3+3\cdot 1)} = \frac{-8}{36} = \frac{-2}{9}$$

55) $$\lim_{x\to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \left[\frac{0}{0}\right] = \lim_{x\to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4}$$

$$\frac{(\sqrt{1+2x}-3)(\sqrt{x}+2)}{x-4} \cdot \frac{\sqrt{1+2x}+3}{\sqrt{1+2x}+3} = \lim_{x\to 4} \frac{(1+2x-9)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)}$$

$$= \lim_{x\to 4} \frac{(-8+2x)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)} = \lim_{x\to 4} \frac{2\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}(\sqrt{1+2x}+3)} = \frac{2\cdot 4}{6} = \frac{4}{3}$$

56) $$\lim_{x\to 9} \frac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}} = \left[\frac{0}{0}\right] = \lim_{x\to 9} \frac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}} \cdot \frac{\sqrt{2x}+3\sqrt{2}}{\sqrt{2x}+3\sqrt{2}} =$$

$$= \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{2x}+3\sqrt{2})}{2x-9\cdot 2} = \lim_{x\to 9} \frac{(\sqrt{x}-3)\cdot(\sqrt{2x}+3\sqrt{2})}{2x-18} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} =$$

$$= \lim_{x\to 9} \frac{(\sqrt{2x}+3\sqrt{2})\cancel{(x-9)}}{2\cancel{(x-9)}(\sqrt{x}+3)} = \frac{\sqrt{18}+3\sqrt{2}}{2\cdot 6} = \frac{3\sqrt{2}+3\sqrt{2}}{12} = \frac{\sqrt{2}}{2}$$

57) $$\lim_{x\to 1} \frac{2-\sqrt{x+3}}{\sqrt{x}-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{2-\sqrt{x+3}}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{(2-\sqrt{x+3})(\sqrt{x}+1)}{x-1}$$

$$= \lim_{x\to 1} \frac{(2-\sqrt{x+3})(\sqrt{x}+1)}{x-1} \cdot \frac{2+\sqrt{x+3}}{2+\sqrt{x+3}} = \lim_{x\to 1} \frac{(4-(x+3))(\sqrt{x}+1)}{(x-1)(2+\sqrt{x+3})} =$$

$$= \lim_{x\to 1} \frac{\cancel{(1-x)}(\sqrt{x}+1)}{-\cancel{(x-1)}(2+\sqrt{x+3})} = \frac{2}{-(2+2)} = -\frac{1}{2}$$