Derivace

- $\left(x^3 - \sqrt[3]{x} - \frac{2}{\sqrt{x}} - 1\right)' = \left(x^3 - x^{\frac{1}{3}} - 2x^{-\frac{1}{2}} - 1\right)' = 3x^2 - \frac{1}{3}x^{-\frac{2}{3}} - 2\cdot\left(-\frac{1}{2}x^{-\frac{3}{2}}\right) =$

  $= 3x^2 - \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{\sqrt{x^3}}$

- $(x^2\cdot \operatorname{cotg} x)' = 2x\cdot \operatorname{cotg} x + x^2\cdot\left(-\frac{1}{\sin^2 x}\right) = x\cdot\left(2\operatorname{cotg} x - \frac{x}{\sin^2 x}\right)$

- $(2^x\cdot x^2)' = 2^x\cdot \ln 2\cdot x^2 + 2^x\cdot 2x = 2^x\cdot x(x\ln 2 + 2)$

- $(\sqrt{x}\cdot \ln x)' = \frac{1}{2}x^{-\frac{1}{2}}\cdot \ln x + \sqrt{x}\cdot\frac{1}{x} = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$

  POZN: $\frac{\sqrt{x}}{x} = \frac{x^{\frac{1}{2}}}{x} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$

- $\left(\frac{x^3-2x}{x+1}\right)' = \frac{(3x^2-2)\cdot(x+1) - (x^3-2x)\cdot 1}{(x+1)^2} = \frac{3x^3-2x+3x^2-2-x^3+2x}{(x+1)^2} =$

  $= \frac{2x^3+3x^2-2}{(x+1)^2}$

- $\left(\frac{e^x}{x+e^x}\right)' = \frac{e^x\cdot(x+e^x) - e^x\cdot(1+e^x)}{(x+e^x)^2} = \frac{xe^x+e^{2x}-e^x-e^{2x}}{(x+e^x)^2} = \frac{e^x(x-1)}{(x+e^x)^2}$

- $\left(\frac{3x+5}{x^2-1}\right)' = \frac{3(x^2-1)-(3x+5)\cdot 2x}{(x^2-1)^2} = \frac{3x^2-3-6x^2-10x}{(x^2-1)^2} = \frac{-3x^2-10x-3}{(x^2-1)^2}$

- $(\ln\sqrt{\sin x})' = \frac{1}{\sqrt{\sin x}}\cdot\frac{1}{2}(\sin x)^{-\frac{1}{2}}\cdot\cos x = \frac{\cos x}{2\cdot\sqrt{\sin x}\cdot\sqrt{\sin x}} = \frac{\cos x}{2\sin x} = \frac{1}{2}\operatorname{cotg} x$

- $\left(\sqrt{x^2-5x+1}\right)' = \frac{1}{2}(x^2-5x+1)^{-\frac{1}{2}}\cdot(2x-5) = \frac{2x-5}{2\sqrt{x^2-5x+1}}$

- $\left(e^{4x^2-5}\right)' = e^{4x^2-5}\cdot 8x$

- $(\operatorname{arctg}(x-1))' = \frac{1}{1+(x-1)^2}\cdot 1 = \frac{1}{1+(x-1)^2}$

- $(\cos^3 x^2)' = 3\cos^2 x^2\cdot(-\sin x^2)\cdot 2x = -6x\cdot\cos^2 x^2\cdot\sin x^2$

- $(\ln^2(x^3-2x))' = 2\ln(x^3-2x)\cdot\frac{1}{x^3-2x}\cdot(3x^2-2)$

- $\left(3^{e^x-2x+x^2}\right)' = 3^{e^x-2x+x^2}\cdot\ln 3\cdot(e^x-2+2x)$

- $$(x^{\cos x})' = (e^{\cos x\cdot \ln x})' = e^{\cos x\cdot\ln x}\cdot\left(-\sin x\cdot \ln x + \cos x\cdot \frac{1}{x}\right) =$$
$$= x^{\cos x}\cdot\left(-\sin x \ln x + \frac{\cos x}{x}\right)$$

toto přepíšu zpět ve výsledku

$$e^{g\cdot\ln f} \Rightarrow f^{g}$$

- $$(x^{\sqrt{x}})' = (e^{\sqrt{x}\ln x})' = e^{\sqrt{x}\ln x}\cdot\left(\frac{1}{2}x^{-\frac{1}{2}}\ln x + \sqrt{x}\cdot\frac{1}{x}\right) = x^{\sqrt{x}}\left(\frac{\ln x}{2\sqrt{x}}+\frac{1}{\sqrt{x}}\right)$$

- $$((\sin x)^{x+2})' = (e^{(x+2)\cdot\ln\sin x})' = e^{(x+2)\ln\sin x}\cdot\left(1\cdot\ln\sin x + (x+2)\cdot\frac{1}{\sin x}\cdot\cos x\right)$$
$$= (\sin x)^{x+2}\left(\ln\sin x + \frac{(x+2)\cos x}{\sin x}\right)$$

- $$(\sqrt[3]{x}^{\,x})' = (e^{x\cdot\ln\sqrt[3]{x}})' = e^{x\cdot\ln\sqrt[3]{x}}\cdot\left(\ln\sqrt[3]{x} + x\cdot\frac{1}{\sqrt[3]{x}}\cdot\frac{1}{3}x^{-\frac{2}{3}}\right) =$$
$$= \sqrt[3]{x}^{\,x}\cdot\left(\ln\sqrt[3]{x} + \frac{\cancel{x}}{\cancel{x^{\frac{1}{3}}}\cdot 3\cdot\cancel{x^{\frac{2}{3}}}}\right) = \sqrt[3]{x}^{\,x}\left(\ln\sqrt[3]{x}+\frac{1}{3}\right)$$

- $$\left(\ln\frac{x+1}{x-3}\right)' = \frac{1}{\frac{x+1}{x-3}}\cdot\frac{1\cdot(x-3)-(x+1)\cdot 1}{(x-3)^2} = \frac{x-3}{x+1}\cdot\frac{-4}{(x-3)^2} = \frac{-4}{(x+1)(x-3)}$$

- $$(x^3\cdot e^{x^2})' = 3x^2\cdot e^{x^2} + x^3\cdot e^{x^2}\cdot 2x = 3x^2e^{x^2}+2x^4e^{x^2}$$

- $$\left(\operatorname{arctg}\frac{\sqrt{x}}{2}\right)' = \frac{1}{1+\frac{x}{4}}\cdot\frac{1}{2}\cdot\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{\frac{4+x}{4}}\cdot\frac{1}{4\sqrt{x}} = \frac{4}{4+x}\cdot\frac{1}{4\sqrt{x}} =$$
$$= \frac{1}{(4+x)\sqrt{x}}$$

- $$((x^2+4x+1)^3\cdot e^{x^2-1})' = 3(x^2+4x+1)^2\cdot(2x+4)\cdot e^{x^2-1} + (x^2+4x+1)^3\cdot e^{x^2-1}\cdot 2x$$
$$= (x^2+4x+1)^2 e^{x^2-1}\cdot\left[6x+12+2x^3+8x^2+2x\right] = (x^2+4x+1)^2\cdot e^{x^2-1}(2x^3+8x^2+8x+12)$$

- $$\left(\frac{x^2\cdot\sin x}{x^2-1}\right)' = \frac{(2x\cdot\sin x + x^2\cdot\cos x)\cdot(x^2-1) - x^2\sin x\cdot 2x}{(x^2-1)^2} = \frac{2x^3\sin x + x^4\cos x - 2x\sin x - x^2\cos x - 2x^3\sin x}{(x^2-1)^2} = \frac{x^4\cos x - 2x\sin x - x^2\cos x}{(x^2-1)^2}$$

- $$\left(\frac{e^x\cdot x^3}{2x-5}\right)' = \frac{(e^x\cdot x^3 + e^x\cdot 3x^2)(2x-5) - e^x x^3\cdot 2}{(2x-5)^2} = \frac{e^x\cdot x^2\left[(x+3)(2x-5)-2x\right]}{(2x-5)^2} = \frac{e^x x^2(2x^2-x-15)}{(2x-5)^2}$$

- $(x\cdot e^{-x+5})' = e^{-x+5} + x\cdot e^{-x+5}(-1) = e^{-x+5}(1-x)$

- $((x^2+5x-3)\cdot e^{x^2})' = (2x+5)e^{x^2} + (x^2+5x-3)\cdot e^{x^2}\cdot 2x =$
  $= e^{x^2}(2x+5+2x^3+10x^2-6x) = e^{x^2}(2x^3+10x^2-4x+5)$

- $(\sqrt[3]{x^2}\cdot \ln x^2)' = \frac{2}{3}\cdot x^{-\frac{1}{3}}\cdot \ln x^2 + \sqrt[3]{x^2}\cdot\frac{1}{x^2}\cdot 2x =$
  $= \frac{2\ln x^2}{3\sqrt[3]{x}} + \frac{\sqrt[3]{x^2}\cdot 2}{x} = \frac{2\ln x^2}{3\cdot\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x}}$

  POZN: $\frac{\sqrt[3]{x^2}}{x} = \frac{x^{\frac{2}{3}}}{x^1} = \frac{1}{x^{\frac{1}{3}}}$

- $\left(\operatorname{arctg}\frac{x-2}{x+1}\right)' = \frac{1}{1+\left(\frac{x-2}{x+1}\right)^2}\cdot\frac{1(x+1)-(x-2)\cdot 1}{(x+1)^2} =$
  $= \frac{1}{\frac{(x+1)^2+(x-2)^2}{(x+1)^2}}\cdot\frac{x+1-x+2}{(x+1)^2} = \frac{(x+1)^2\cdot 3}{[(x+1)^2+(x-2)^2]\cdot(x+1)^2} = \frac{3}{(x+1)^2+(x-2)^2}$

- $(\sqrt{x^2-5x}\cdot e^{-x})' = \frac{1}{2}(x^2-5x)^{-\frac{1}{2}}\cdot(2x-5)\cdot e^{-x} + \sqrt{x^2-5x}\cdot e^{-x}(-1) =$
  $= \frac{(2x-5)}{2\cdot\sqrt{x^2-5x}\cdot e^{x}} - \frac{\sqrt{x^2-5x}}{e^{x}}$

- $\left(\frac{(2x-4)^3}{x^2-6}\right)' = \frac{3(2x-4)^2\cdot 2\cdot(x^2-6)-(2x-4)^3\cdot 2x}{(x^2-6)^2} =$
  $= \frac{(2x-4)^2\cdot 2\cdot[3x^2-18-4x^2+8x]}{(x^2-6)^2} = \frac{(2x-4)^2\cdot 2\cdot(-x^2+8x-18)}{(x^2-6)^2}$

- $\left(x^4\cdot\ln\frac{x}{e^x}\right)' = 4x^3\cdot\ln\frac{x}{e^x} + x^4\cdot\frac{1}{\frac{x}{e^x}}\cdot\frac{1\cdot e^x - x\cdot e^x}{(e^x)^2} =$
  $= 4x^3\cdot\ln\frac{x}{e^x} + x^4\cdot\frac{\cancel{e^x}}{x}\cdot\frac{e^x(1-x)}{\cancel{e^{2x}}} = 4x^3\ln\frac{x}{e^x} + x^3(1-x)$

- $(\sqrt{x^2-5x+1}-x+1)' = \frac{1}{2}(x^2-5x+1)^{-\frac{1}{2}}\cdot(2x-5)-1 =$
  $= \frac{2x-5}{2\sqrt{x^2-5x+1}} - 1$

- $\left(\sqrt{\frac{3x-1}{x+2}}\right)' = \frac{1}{2}\left(\frac{3x-1}{x+2}\right)^{-\frac{1}{2}}\cdot\frac{3(x+2)-(3x-1)}{(x+2)^2} = \frac{1}{2\sqrt{\frac{3x-1}{x+2}}}\cdot\frac{3x+6-3x+1}{(x+2)^2} = \frac{\sqrt{x+2}}{2\sqrt{3x-1}}\cdot\frac{7}{(x+2)^2}$