

Derivace

$$\begin{aligned} \bullet (x^3 - \sqrt[3]{x} - \frac{2}{\sqrt{x}} - 1)' &= (x^3 - x^{\frac{1}{3}} - 2x^{-\frac{1}{2}} - 1)' = 3x^2 - \frac{1}{3}x^{-\frac{2}{3}} - 2(-\frac{1}{2}x^{-\frac{3}{2}}) = \\ &= 3x^2 - \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{\sqrt{x^3}} \end{aligned}$$

$$\bullet (x^2 \cdot \cot x)' = 2x \cdot \cot x + x^2 \cdot \left(-\frac{1}{\sin^2 x}\right) = x \cdot \left(2 \cot x - \frac{x}{\sin^2 x}\right)$$

$$\bullet (2^x \cdot x^2)' = 2^x \cdot \ln 2 \cdot x^2 + 2^x \cdot 2x = 2^x \cdot x(x \ln 2 + 2)$$

$$\bullet (\sqrt{x} \cdot \ln x)' = \frac{1}{2}x^{-\frac{1}{2}} \cdot \ln x + \sqrt{x} \cdot \frac{1}{x} = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \quad \text{POZN: } \frac{\sqrt{x}}{x} = x^{\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

$$\begin{aligned} \bullet \left(\frac{x^3 - 2x}{x+1}\right)' &= \frac{(3x^2 - 2) \cdot (x+1) - (x^3 - 2x) \cdot 1}{(x+1)^2} = \frac{3x^3 - 2x + 3x^2 - 2 - x^3 + 2x}{(x+1)^2} = \\ &= \frac{2x^3 + 3x^2 - 2}{(x+1)^2} \end{aligned}$$

$$\bullet \left(\frac{e^x}{x+e^x}\right)' = \frac{e^x \cdot (x+e^x) - e^x \cdot (1+e^x)}{(x+e^x)^2} = \frac{xe^x + e^{2x} - e^x - e^{2x}}{(x+e^x)^2} = \frac{e^x(x-1)}{(x+e^x)^2}$$

$$\bullet \left(\frac{3x+5}{x^2-1}\right)' = \frac{3(x^2-1) - (3x+5) \cdot 2x}{(x^2-1)^2} = \frac{3x^2 - 3 - 6x^2 - 10x}{(x^2-1)^2} = \frac{-3x^2 - 10x - 3}{(x^2-1)^2}$$

$$\bullet (\ln \sqrt{\sin x})' = \frac{1}{\sqrt{\sin x}} \cdot \frac{1}{2}(\sin x)^{-\frac{1}{2}} \cdot \cos x = \frac{\cos x}{2\sqrt{\sin x} \cdot \sqrt{\sin x}} = \frac{\cos x}{2\sin x} = \frac{1}{2} \cot x$$

$$\bullet (\sqrt{x^2 - 5x + 1})' = \frac{1}{2}(x^2 - 5x + 1)^{-\frac{1}{2}} \cdot (2x - 5) = \frac{2x - 5}{2\sqrt{x^2 - 5x + 1}}$$

$$\bullet (e^{4x^2-5})' = e^{4x^2-5} \cdot 8x$$

$$\bullet (\arctg(x-1))' = \frac{1}{1+(x-1)^2} \cdot 1 = \frac{1}{1+(x-1)^2}$$

$$\bullet (\cos^3 x^2)' = 3\cos^2 x^2 \cdot (-\sin x^2) \cdot 2x = -6x \cdot \cos^2 x^2 \cdot \sin x^2$$

$$\bullet (\ln^2(x^3 - 2x))' = 2\ln(x^3 - 2x) \cdot \frac{1}{x^3 - 2x} \cdot (3x^2 - 2)$$

$$\bullet (3^{e^x - 2x + x^2})' = 3^{e^x - 2x + x^2} \cdot \ln 3 \cdot (e^x - 2 + 2x)$$

- $(\sin \sqrt{x})' = \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$
- $(\cot g x^4)' = -\frac{1}{\sin^2 x^4} \cdot 4x^3$
- $(2 \arcsin(\ln x))' = 2 \cdot \frac{1}{\sqrt{1-\ln^2 x}} \cdot \frac{1}{x}$
- $(3^{\sqrt{x}})' = 3^{\sqrt{x}} \cdot \ln 3 \cdot \frac{1}{2\sqrt{x}}$
- $(e^{\cos x})' = e^{\cos x} \cdot (-\sin x)$
- $(\arctg(\sin x))' = \frac{1}{1+\sin^2 x} \cdot \cos x$
- $(\sqrt{x^3+5x})' = \frac{1}{2\sqrt{x^3+5x}} \cdot (3x^2+5)$
- $(\operatorname{Ag}^2 x)' = 2 \operatorname{Ag} x \cdot \frac{1}{\cos^2 x}$
- $(\operatorname{Ag} x^2)' = \frac{1}{\cos^2 x^2} \cdot 2x$
- $(e^{x \cdot \sin x})' = e^{x \sin x} \cdot (\sin x + x \cos x)$
- $(\sin x \cdot e^{x^2})' = \cos x \cdot e^{x^2} + \sin x \cdot e^{x^2} \cdot 2x$
- $(\ln x^2 \cdot \sin x)' = \frac{1}{x^2} \cdot 2x \cdot \sin x + \ln x^2 \cdot \cos x = \frac{2}{x} \sin x + \ln x^2 \cdot \cos x$
- $\left(\frac{\cos^2 x}{x^3+2x}\right)' = \frac{2\cos x \cdot (-\sin x) \cdot (x^3+2x) - \cos^2 x \cdot (3x^2+2)}{(x^3+2x)^2}$
- $\left(\frac{e^{x^2}}{\sin 3x}\right)' = \frac{e^{x^2} \cdot 2x \cdot \sin 3x - e^{x^2} \cdot \cos 3x \cdot 3}{(\sin 3x)^2}$
- $\left(\frac{\ln 5x}{2x^3-2}\right)' = \frac{\frac{1}{5x} \cdot 5 \cdot (2x^3-2) - \ln 5x \cdot (6x^2)}{(2x^3-2)^2}$
- $(\sqrt{\cos 4x})' = \frac{1}{2\sqrt{\cos 4x}} \cdot (-\sin 4x) \cdot 4 = \frac{-2 \sin 4x}{\sqrt{\cos 4x}}$
- $\left(\ln \frac{x+1}{3x}\right)' = \frac{1}{\frac{x+1}{3x}} \cdot \frac{1 \cdot 3x - (x+1) \cdot 3}{9x^2} = \frac{3x-3x-3}{(x+1) \cdot 3x} = \frac{-1}{x(x+1)}$

- $(\arccos \sqrt{x} \cdot \sin x)' = \frac{-1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot \sin x + \arccos \sqrt{x} \cdot \cos x$
- $(\cos \frac{1}{x})' = \frac{1}{\sin^2 \frac{1}{x}} \cdot (-x^{-2}) = \frac{1}{x^2 \sin^2 \frac{1}{x}}$
- $(\sin \sqrt[3]{x})' = \cos \sqrt[3]{x} \cdot \frac{1}{3\sqrt[3]{x^2}}$
- $(\sqrt{\ln x} \cdot e^{3x})' = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} \cdot e^{3x} + \sqrt{\ln x} \cdot e^{3x} \cdot 3$
- $(\frac{\cos 5x}{\ln 3x})' = \frac{-\sin 5x \cdot 5 \cdot \ln 3x - \cos 5x \cdot \frac{1}{3x} \cdot 3}{\ln^2 3x}$
- $(\lg \frac{e^x}{x})' = \frac{1}{\cos^2 \frac{e^x}{x}} \cdot \frac{e^x \cdot x - e^x}{x^2}$
- $(\arctan^2 x^7)' = 2 \arctan x^7 \cdot \frac{1}{1+x^{14}} \cdot 7x^6$

$$\bullet \left(\ln \frac{x+1}{x-3} \right)' = \frac{1}{\frac{x+1}{x-3}} \cdot \frac{1 \cdot (x-3) - (x+1) \cdot 1}{(x-3)^2} = \frac{x-3}{x+1} \cdot \frac{-4}{(x-3)^2} = \frac{-4}{(x+1)(x-3)}$$

$$\bullet (x^3 \cdot e^{x^2})' = 3x^2 \cdot e^{x^2} + x^3 \cdot e^{x^2} \cdot 2x = 3x^2 e^{x^2} + 2x^4 e^{x^2}$$

$$\bullet \left(\arctan \frac{\sqrt{x}}{2} \right)' = \frac{1}{1 + \frac{x}{4}} \cdot \frac{1}{2} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\frac{4+x}{4}} \cdot \frac{1}{4\sqrt{x}} = \frac{4}{4+x} \cdot \frac{1}{4\sqrt{x}} = \frac{1}{(4+x)\sqrt{x}}$$

$$\bullet \left((x^2+4x+1)^3 \cdot e^{x^2-1} \right)' = 3(x^2+4x+1)^2 \cdot (2x+4) \cdot e^{x^2-1} + (x^2+4x+1)^3 \cdot e^{x^2-1} \cdot 2x$$

$$= (x^2+4x+1)^2 e^{x^2-1} \cdot [6x+12+2x^3+8x^2+2x] = (x^2+4x+1)^2 \cdot e^{x^2-1} (2x^3+8x^2+8x+12)$$

$$\bullet \left(\frac{x^2 \cdot \sin x}{x^2-1} \right)' = \frac{(2x \cdot \sin x + x^2 \cdot \cos x) \cdot (x^2-1) - x^2 \sin x \cdot 2x}{(x^2-1)^2} = \frac{2x^3 \sin x + x^4 \cos x - 2x \sin x - x^2 \cos x - 2x^3 \sin x}{(x^2-1)^2} = \frac{x^4 \cos x - 2x \sin x - x^2 \cos x}{(x^2-1)^2}$$

$$\bullet \left(\frac{e^x \cdot x^3}{2x-5} \right)' = \frac{(e^x \cdot x^3 + e^x \cdot 3x^2)(2x-5) - e^x x^3 \cdot 2}{(2x-5)^2} = \frac{e^x \cdot x^2 [(x+3)(2x-5) - 2x]}{(2x-5)^2} = \frac{e^x x^2 (2x^2 - x - 15)}{(2x-5)^2}$$

$$\cdot (x \cdot e^{-x+5})' = e^{-x+5} + x \cdot e^{-x+5}(-1) = e^{-x+5}(1-x)$$

$$\cdot ((x^2+5x-3) \cdot e^{x^2})' = (2x+5)e^{x^2} + (x^2+5x-3) \cdot e^{x^2} \cdot 2x = \\ = e^{x^2}(2x+5+2x^3+10x^2-6x) = e^{x^2}(2x^3+10x^2-4x+5)$$

$$\cdot (\sqrt[3]{x^2} \cdot \ln x^2)' = \frac{2}{3} \cdot x^{-\frac{1}{3}} \cdot \ln x^2 + \sqrt[3]{x^2} \cdot \frac{1}{x^2} \cdot 2x = \\ = \frac{2 \ln x^2}{3 \sqrt[3]{x}} + \frac{\sqrt[3]{x^2} \cdot 2}{x} = \frac{2 \ln x^2}{3 \sqrt[3]{x}} + \frac{2}{\sqrt[3]{x}}$$

POZN:

$$\frac{\sqrt[3]{x^2}}{x} = \frac{x^{\frac{2}{3}}}{x^1} = \frac{1}{x^{\frac{1}{3}}}$$

$$\cdot \left(\arctg \frac{x-2}{x+1} \right)' = \frac{1}{1 + \left(\frac{x-2}{x+1} \right)^2} \cdot \frac{1(x+1) - (x-2) \cdot 1}{(x+1)^2} = \\ = \frac{1}{\frac{(x+1)^2 + (x-2)^2}{(x+1)^2}} \cdot \frac{x+1-x+2}{(x+1)^2} = \frac{(x+1)^2 \cdot 3}{[(x+1)^2 + (x-2)^2] \cdot (x+1)^2} = \frac{3}{(x+1)^2 + (x-2)^2}$$

$$\cdot (\sqrt{x^2-5x} \cdot e^{-x})' = \frac{1}{2} (x^2-5x)^{-\frac{1}{2}} \cdot (2x-5) \cdot e^{-x} + \sqrt{x^2-5x} \cdot e^{-x}(-1) = \\ = \frac{(2x-5)}{2 \cdot \sqrt{x^2-5x} \cdot e^x} - \frac{\sqrt{x^2-5x}}{e^x}$$

$$\cdot \left(\frac{(2x-4)^3}{x^2-6} \right)' = \frac{3(2x-4)^2 \cdot 2 \cdot (x^2-6) - (2x-4)^3 \cdot 2x}{(x^2-6)^2} = \\ = \frac{(2x-4)^2 \cdot 2 \cdot [3x^2 - 18 - 4x^2 + 8x]}{(x^2-6)^2} = \frac{(2x-4)^2 \cdot 2 \cdot (-x^2 + 8x - 18)}{(x^2-6)^2}$$

$$\cdot \left(x^4 \cdot \ln \frac{x}{e^x} \right)' = 4x^3 \cdot \ln \frac{x}{e^x} + x^4 \cdot \frac{1}{\frac{x}{e^x}} \cdot \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2} = \\ = 4x^3 \cdot \ln \frac{x}{e^x} + x^4 \cdot \frac{e^x}{x} \cdot \frac{e^x(1-x)}{e^{2x}} = 4x^3 \ln \frac{x}{e^x} + x^3(1-x)$$

$$\cdot (\sqrt{x^2-5x+1} - x + 1)' = \frac{1}{2} (x^2-5x+1)^{-\frac{1}{2}} \cdot (2x-5) - 1 = \\ = \frac{2x-5}{2\sqrt{x^2-5x+1}} - 1$$

$$\cdot \left(\sqrt{\frac{3x-1}{x+2}} \right)' = \frac{1}{2} \left(\frac{3x-1}{x+2} \right)^{-\frac{1}{2}} \cdot \frac{3(x+2) - (3x-1)}{(x+2)^2} = \frac{1}{2\sqrt{\frac{3x-1}{x+2}}} \cdot \frac{3x+6-3x+1}{(x+2)^2} = \frac{\sqrt{x+2}}{2\sqrt{3x-1}} \cdot \frac{7}{(x+2)^2}$$