

Derivujte zadané funkce.

Differentiate the following functions.

Př.  $y = \frac{x}{x^2 + 1}$

$$\begin{aligned} y' &= \frac{(x)' \cdot (x^2 + 1) - x \cdot (x^2 + 1)'}{(x^2 + 1)^2} \\ &= \frac{(x^2 + 1) - x \cdot 2x}{(x^2 + 1)^2} \\ &= \frac{1 - x^2}{(1 + x^2)^2} \end{aligned}$$

Př.  $y = e^{\arctg x^2}$

$$\begin{aligned} y' &= e^{\arctg x^2} \cdot (\arctg x^2)' \\ &= e^{\arctg x^2} \cdot \frac{1}{1 + (x^2)^2} \cdot (x^2)' \\ &= \frac{2xe^{\arctg x^2}}{1 + x^4} \end{aligned}$$

Př.  $y = x \ln^2 x$

$$\begin{aligned} y' &= (x)' \cdot \ln^2 x + x \cdot (\ln^2 x)' \\ &= 1 \cdot \ln^2 x + x \cdot 2 \ln x \cdot (\ln x)' \\ &= \ln^2 x + x \cdot 2 \ln x \cdot \frac{1}{x} \\ &= (2 + \ln x) \ln x \end{aligned}$$

Př.  $y = \frac{1 - x^3}{x^2}$

$$\begin{aligned} y' &= \frac{(1 - x^3)'x^2 - (1 - x^3) \cdot (x^2)'}{(x^2)^2} \\ &= \frac{-3x^2x^2 - (1 - x^3)2x}{(x^2)^2} \\ &= \frac{-3x^4 - 2x + 2x^4}{x^4} \\ &= -\frac{2 + x^3}{x^3} \end{aligned}$$

Př.  $y = \left(\frac{x-1}{x+1}\right)^2$

$$\begin{aligned} y' &= 2 \frac{x-1}{x+1} \left(\frac{x-1}{x+1}\right)' \\ &= 2 \frac{x-1}{x+1} \cdot \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2} \\ &= 2 \frac{x-1}{x+1} \cdot \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2} \\ &= 2 \frac{x-1}{x+1} \cdot \frac{2}{(x+1)^2} \\ &= 4 \frac{x-1}{(x+1)^3} \end{aligned}$$

Př.  $y = e^x(x^2 - 2x + 2)$

$$\begin{aligned} y' &= (e^x)'(x^2 - 2x + 2) + e^x(x^2 - 2x + 2)' \\ &= e^x(x^2 - 2x + 2) + e^x(2x - 2) \\ &= e^x(x^2 - 2x + 2 + 2x - 2) \\ &= e^x x^2 \end{aligned}$$

Př.  $y = \sqrt[3]{\frac{1+x^3}{1-x^3}}$

$$\begin{aligned} y' &= \frac{1}{3} \left(\frac{1+x^3}{1-x^3}\right)^{-2/3} \left(\frac{1+x^3}{1-x^3}\right)' \\ &= \frac{1}{3} \left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{(1+x^3)'(1-x^3) - (1+x^3)(1-x^3)'}{(1-x^3)^2} \\ &= \frac{1}{3} \left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{3x^2(1-x^3) - (1+x^3)(-3x^2)}{(1-x^3)^2} \\ &= \frac{1}{3} \left(\frac{1-x^3}{1+x^3}\right)^{2/3} \cdot \frac{6x^2}{(1-x^3)^2} \\ &= \sqrt[3]{\frac{1+x^3}{1-x^3}} \frac{1-x^3}{1+x^3} \cdot \frac{2x^2}{(1-x^3)^2} \\ &= \sqrt[3]{\frac{1+x^3}{1-x^3}} \frac{2x^2}{1-x^6} \end{aligned}$$

Př.  $y = x \ln(x^2 - 1)$

$$\begin{aligned} y' &= x' \cdot \ln(x^2 - 1) + x \cdot (\ln(x^2 - 1))' \\ &= 1 \cdot \ln(x^2 - 1) + x \cdot \frac{1}{x^2 - 1} \cdot (x^2 - 1)' \\ &= \ln(x^2 - 1) + x \cdot \frac{1}{x^2 - 1} \cdot 2x \\ &= \ln(x^2 - 1) + \frac{2x^2}{x^2 - 1} \end{aligned}$$

$$\text{Př. } y = \frac{1}{4} \ln \frac{x^2 - 1}{x^2 + 1}$$

$$\begin{aligned} y' &= \frac{1}{4} \frac{x^2 + 1}{x^2 - 1} \frac{2x(x^2 + 1) - (x^2 - 1)2x}{(x^2 + 1)^2} \\ &= \frac{x^2 + 1}{x^2 - 1} \frac{4x}{(x^2 + 1)^2} \\ &= \frac{x}{(x^2 - 1)(x^2 + 1)} \end{aligned}$$

$$\text{Př. } y = \sqrt{x+1} - \ln(1 + \sqrt{x+1})$$

$$\begin{aligned} y' &= \frac{1}{2\sqrt{x+1}} - \frac{1}{1 + \sqrt{x+1}} \cdot \frac{1}{2\sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}} \left( 1 - \frac{1}{1 + \sqrt{x+1}} \right) \\ &= \frac{1}{2\sqrt{x+1}} \frac{\sqrt{x+1}}{1 + \sqrt{x+1}} \\ &= \frac{1}{2(1 + \sqrt{x+1})} \end{aligned}$$

$$\text{Př. } y = \frac{\sin x - x \cos x}{\cos x + x \sin x}$$

$$\begin{aligned} y' &= \frac{[\cos x - (\cos x - x \sin x)](\cos x + x \sin x) - (\sin x - x \cos x)[- \sin x + \sin x + x \cos x]}{(\cos x + x \sin x)^2} \\ &= \frac{x \sin x \cos x + x^2 \sin^2 x - [x \sin x \cos x - x^2 \cos^2 x]}{(\cos x + x \sin x)^2} \\ &= \frac{x^2}{(\cos x + x \sin x)^2} \end{aligned}$$

$$\text{Př. } y = \sqrt{1-x} \cdot \arcsin \sqrt{x}$$

$$\begin{aligned} y' &= (\sqrt{1-x})' \cdot \arcsin \sqrt{x} + \sqrt{1-x} \cdot (\arcsin \sqrt{x})' \\ &= \frac{1}{2\sqrt{1-x}} \cdot (1-x)' \cdot \arcsin \sqrt{x} + \sqrt{1-x} \cdot \frac{1}{\sqrt{1-(\sqrt{x})^2}} (\sqrt{x})' \\ &= -\frac{1}{2\sqrt{1-x}} \arcsin \sqrt{x} + \sqrt{1-x} \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} - \frac{\arcsin \sqrt{x}}{2\sqrt{1-x}} \end{aligned}$$

$$1. y = -\frac{1}{9}x^4 + \frac{2}{3}x^2$$

$$2. y = 4x^3 - 3x^4$$

$$3. y = -2 + 12x - x^3$$

$$4. y = x + \frac{4}{x}$$

$$5. y = \frac{x}{(x+1)^2}$$

$$6. y = x^2 - 2 \ln x$$

$$7. y = (3-x)\sqrt{x}$$

$$8. y = 2\sqrt{x} - x$$

$$9. y = \frac{x^2}{1-x}$$

$$10. y = 1 + x^2 - \frac{x^4}{4}$$

$$11. y = \frac{x-2}{\sqrt{x^2+1}}$$

$$12. y = \frac{x^2}{x^2+1}$$

$$13. y = \left( \frac{1+x}{1-x} \right)^2$$

$$14. y = \left( \frac{1+x}{1-x} \right)^4$$

$$15. y = \sqrt[3]{2x^2 - x^3}$$

$$16. y = \frac{x}{1+x^2}$$

$$17. y = \frac{1-x^3}{x^2}$$

$$18. y = \frac{1+x^2}{1-x^2} = -1 + \frac{2}{1-x^2}$$

19.  $y = \frac{\ln^2 x}{x}$
20.  $y = \frac{\ln x}{\sqrt{x}}$
21.  $y = \frac{e^x}{1+x}$
22.  $y = x^{\frac{2}{3}} e^{-x}$
23.  $y = x^2 e^{-x}$
24.  $y = x e^{\frac{1}{x}}$
25.  $y = x - \ln(1+x)$
26.  $y = x - \ln(1+x^2)$
27.  $y = e^{-x} \sin x$
28.  $y = x + \frac{2x}{1+x^2}$
29.  $y = \frac{x^2}{2} + \frac{8}{x^3}$
30.  $y = \sqrt{2x-x^2}$
31.  $y = (x+1)^{10} e^{-x}$
32.  $y = \frac{x^2}{2^x}$
33.  $y = \arcsin(\sqrt{x})$
34.  $y = x \arcsin(\sqrt{x})$
35.  $y = \frac{\arctan(x)}{x^2+1}$
36.  $y = \sin(\ln(1+\cos(x)))$
37.  $y = \ln\left(\frac{\cos(x)}{(\sin(x))^2}\right)$
38.  $y = \sqrt{\frac{x^2+1}{x^2-1}}$
39.  $y = x^2 \ln(x^2+x+1)$
40.  $y = (x^2+1) \arctan(\sqrt{1-x^2})$
41.  $y = (x^2+3x+6) e^{1-x}$
42.  $y = (x^3+2x-1) \sin(2x)$
43.  $y = \frac{x e^{-x}}{x+1}$
44.  $y = \frac{x^2+1}{\ln(x)}$
45.  $y = \ln\left(\frac{1+\sin(x)}{1-\sin(x)}\right)$
46.  $y = (x^2-2) \sqrt{x^2+1}$
47.  $y = \ln(1-\sqrt{x+1})$
48.  $y = \arcsin(x\sqrt{1-x})$
49.  $y = \frac{\arcsin(x)}{x^2+1}$
50.  $y = \ln(\sqrt{e^x-x^2})$
51.  $y = \ln(\sqrt{e^{-x}-4x^2})$
52.  $y = \arctan(\ln(1+\sin(x)))$
53.  $y = \sin(\ln(x+\sin(2x)))$
54.  $y = x^2 e^{-\sin(x)}$
55.  $y = \arctan(\sqrt{\ln(x^2-1)})$
56.  $y = x^2 \ln(\arctan(2x-1))$
57.  $y = \frac{\ln(x^3+1)}{x}$
58.  $y = \frac{x^2+1}{\sqrt{2x-1}}$
59.  $y = \frac{\sqrt{x^2+x+1}}{x+1}$
60.  $y = \sqrt{\arctan(x^{-1})}$
61.  $y = \arctan(1/3\sqrt{6-3x})$
62.  $y = (x^2+1) \arctan(x^{-1})$
63.  $y = \ln(x\sqrt{x^2+1})$
64.  $y = (x^2+x+1) \sin(x) + x \cos(x)$
65.  $y = \arctan(\sqrt{2}\sqrt{x-1})$
66.  $y = \frac{\sin(x) - \cos(x)}{x}$
67.  $y = \frac{1+\cos(x)}{\sin(2x)}$
68.  $y = (1-x^2) \arccos(x)$

Výsledky (výsledky jsou u prvních příkladů navíc ještě většinou rozloženy na součin), *solutions*:

- 1:  $-\frac{4}{9}x(x^2-3)$ , 2:  $12x^2(1-x)$ , 3:  $3(2-x)(2+x)$ , 4:  $\frac{(x-2)(x+2)}{x^2}$ , 5:  $\frac{1-x}{(x+1)^3}$ , 6:  $2\frac{(x-1)(x+1)}{x}$ ,  
 7:  $\frac{3}{2\sqrt{x}}(1-x)$ , 8:  $\frac{1-\sqrt{x}}{\sqrt{x}}$ , 9:  $\frac{x(2-x)}{(1-x)^2}$ , 10:  $-2x(x-1)(x+1)$ , 11:  $\frac{2x+1}{(x^2+1)^{\frac{3}{2}}}$ , 12:  $\frac{2x}{(1+x^2)^2}$ ,

- 13:**  $-4 \frac{x+1}{(x-1)^3}$ , **14:**  $-8 \frac{(x+1)^3}{(x-1)^5}$ , **15:**  $\frac{1}{3} \frac{4-3x}{\sqrt[3]{x(2-x)^2}}$ , **16:**  $\frac{1-x^2}{(1+x^2)^2}$ , **17:**  $-\frac{x^3+2}{x^3}$ , **18:**  $\frac{4x}{(1-x^2)^2}$ ,  
**19:**  $\frac{\ln x(2-\ln x)}{x^2}$ , **20:**  $\frac{2-\ln x}{2x^{\frac{3}{2}}}$ , **21:**  $\frac{xe^x}{(x+1)^2}$ , **22:**  $e^{-x} \frac{2-3x}{3\sqrt[3]{x}}$ , **23:**  $e^{-x}x(2-x)$ , **24:**  $e^{\frac{1}{x}} \frac{x-1}{x}$ ,  
**25:**  $\frac{x^2+x-1}{x+1}$ , **26:**  $\frac{(x-1)^2}{x^2+1}$ , **27:**  $e^{-x}(\cos x - \sin x)$ , **28:**  $\frac{x^4+3}{(x^2+1)^2}$ , **29:**  $\frac{x^5-24}{x^4}$ , **30:**  $\frac{1-x}{\sqrt{x(2-x)}}$ ,  
**31:**  $e^{-x}(x+1)^9(9-x)$ , **32:**  $\frac{x(2-x \ln 2)}{2^x}$ , **33:**  $1/2 \frac{1}{\sqrt{x}\sqrt{1-x}}$ , **34:**  $\arcsin(\sqrt{x}) + 1/2 \frac{\sqrt{x}}{\sqrt{1-x}}$ ,  
**35:**  $-\frac{-1+2 \arctan(x)x}{(x^2+1)^2}$ , **36:**  $-\frac{\cos(\ln(1+\cos(x))) \sin(x)}{1+\cos(x)}$ , **37:**  $-\frac{(\cos(x))^2+1}{\sin(x) \cos(x)}$ ,  
**38:**  $-2x \frac{1}{\sqrt{\frac{x^2+1}{x^2-1}}} (x^2-1)^{-2}$ , **39:**  $2x \ln(x^2+x+1) + \frac{x^2(1+2x)}{x^2+x+1}$ ,  
**40:**  $2x \arctan(\sqrt{1-x^2}) - \frac{(x^2+1)x}{\sqrt{1-x^2}(2-x^2)}$ , **41:**  $-e^{1-x}(x+3+x^2)$ ,  
**42:**  $(3x^2+2) \sin(2x) + 2(x^3+2x-1) \cos(2x)$ , **43:**  $-\frac{e^{-x}(-1+x^2+x)}{(x+1)^2}$ , **44:**  $2 \frac{x}{\ln(x)} - \frac{x^2+1}{(\ln(x))^2 x}$ ,  
**45:**  $2(\cos(x))^{-1}$ , **46:**  $3 \frac{x^3}{\sqrt{x^2+1}}$ , **47:**  $1/2 \frac{1}{\sqrt{x+1}(-1+\sqrt{x+1})}$ , **48:**  $-1/2 \frac{-2+3x}{\sqrt{1-x}\sqrt{1-x^2+x^3}}$ ,  
**49:**  $\frac{1}{\sqrt{1-x^2}(x^2+1)} - 2 \frac{\arcsin(x)x}{(x^2+1)^2}$ , **50:**  $1/2 \frac{e^x-2x}{e^x-x^2}$ , **51:**  $1/2 \frac{-e^{-x}-8x}{e^{-x}-4x^2}$ ,  
**52:**  $\frac{\cos(x)}{(1+\sin(x))(1+(\ln(1+\sin(x))))^2}$ , **53:**  $\frac{\cos(\ln(x+\sin(2x)))(1+2\cos(2x))}{x+\sin(2x)}$ ,  
**54:**  $2xe^{-\sin(x)} - x^2 \cos(x)e^{-\sin(x)}$ , **55:**  $\frac{x}{\sqrt{\ln(x^2-1)}(x^2-1)(1+\ln(x^2-1))}$ ,  
**56:**  $2x \ln(\arctan(2x-1)) + 2 \frac{x^2}{(1+(2x-1)^2) \arctan(2x-1)}$ , **57:**  $3 \frac{x}{x^3+1} - \frac{\ln(x^3+1)}{x^2}$ ,  
**58:**  $\frac{3x^2-2x-1}{(2x-1)^{3/2}}$ , **59:**  $1/2 \frac{-1+x}{\sqrt{x^2+x+1}(x+1)^2}$ , **60:**  $-1/2 \frac{1}{\sqrt{\arctan(x^{-1})}(x^2+1)}$ ,  
**61:**  $3/2 \frac{1}{\sqrt{6-3x}(-5+x)}$ , **62:**  $2x \arctan(x^{-1}) - 1$ , **63:**  $\frac{2x^2+1}{(x^2+1)x}$ ,  
**64:**  $\sin(x) + x \sin(x) + \cos(x)x^2 + x \cos(x) + 2 \cos(x)$ , **65:**  $-1/2 \frac{\sqrt{2}}{\sqrt{x^{-1}x}(x+2)}$ ,  
**66:**  $\frac{x \cos(x) + x \sin(x) - \sin(x) + \cos(x)}{x^2}$ , **67:**  $-\frac{\sin(x)}{\sin(2x)} - 2 \frac{(1+\cos(x)) \cos(2x)}{(\sin(2x))^2}$ ,  
**68:**  $-2x \arccos(x) - \sqrt{1-x^2}$  (bez záruky, no warranty)