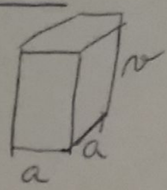
ŘEŠENÍ:

BAZÉN



$V = 256\,000\,l = 256\,000\,dm^3 = 256\,m^3$

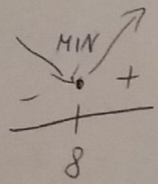
Povrch stěn + dna $\rightarrow$ MIN $\quad S = a^2 + 4\cdot av$

$$V = a^2 \cdot v$$
$$256 = a^2 \cdot v$$
$$\frac{256}{a^2} = v \quad \longrightarrow \quad S = a^2 + \frac{4a\cdot 256}{a^2} = a^2 + \frac{1024}{a}$$

$$S' = 2a + 1024\cdot(-a^{-2}) =$$
$$= 2a - \frac{1024}{a^2}$$

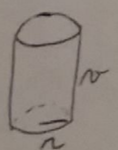
$$2a - \frac{1024}{a^2} = 0$$
$$2a^3 - 1024 = 0$$
$$a^3 = 512$$
$$a = 8$$

$$v = \frac{256}{8^2} = \frac{256}{64} = 4$$



Rozměry dna 8×8 m a výška 4 m.

PARNÍ KOTEL



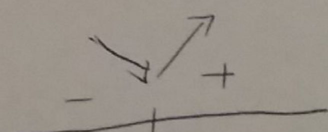
V - dáno

S → min.

$$V = \pi r^2 \cdot v \quad \longrightarrow \quad \frac{V}{\pi r^2} = v$$
$$S = 2\pi r v + 2\pi r^2$$
$$S = \frac{2\pi \cancel{r} \cdot V}{\pi r^2} + 2\pi r^2 = \frac{2V}{r} + 2\pi r^2$$

(r je neznámá, ostatní jsou konstanty)

$$S' = 2V\cdot(-r^{-2}) + 4\pi r = -\frac{2V}{r^2} + 4\pi r$$
$$-\frac{2V}{r^2} + 4\pi r = 0$$
$$-2V + 4\pi r^3 = 0$$
$$r^3 = \frac{2V}{4\pi}$$
$$\boxed{r = \sqrt[3]{\frac{V}{2\pi}}} \qquad v = \frac{V}{\pi\cdot\sqrt[3]{\frac{V^2}{4\pi^2}}} = \boxed{\sqrt[3]{\frac{4V}{\pi}}}$$

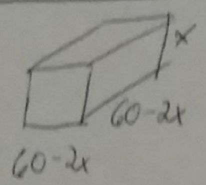


$$\sqrt[3]{\frac{V}{2\pi}}$$

Že se jedná o minimum lze ověřit druhou derivací: $S'' = 2\cdot 2V\cdot r^{-3} + 4\pi = \frac{4V}{r^3} + 4\pi > 0 \rightarrow$ funkce je konvexní

(tedy jedná se o MIN)

KRABICE

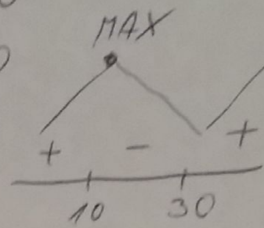


$\max V$

$$V=(60-2x)^2\cdot x=(3600-240x+4x^2)\cdot x$$
$$=3600x-240x^2+4x^3$$

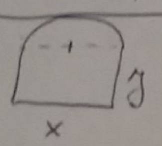
$$V'=3600-480x+12x^2$$

$$12x^2-480x+3600=0$$
$$x^2-40x+300=0$$
$$(x-30)(x-10)=0$$
$$x=30 \quad \boxed{x=10}$$



$$V=(60-2\cdot 10)^2\cdot 10$$
$$=\underline{\underline{16000\,cm^3}}$$

RÁM OKNA



$O=200$

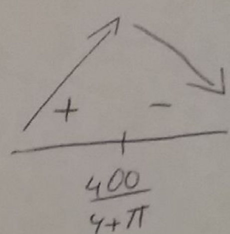
$\max S$

$$S=x\cdot y+\pi\cdot\left(\frac{x}{2}\right)^2\cdot\frac{1}{2}$$
$$=x\cdot y+\frac{\pi x^2}{8}$$

$$O=2y+x+\pi\cdot\frac{x}{2}$$
$$200=2y+x+\frac{\pi x}{2}$$
$$200-x-\frac{\pi x}{2}=2y$$
$$100-\frac{x}{2}-\frac{\pi x}{4}=y$$

$$S=x\cdot\left(100-\frac{x}{2}-\frac{\pi x}{4}\right)+\frac{\pi x^2}{8}$$
$$=100x-\frac{x^2}{2}-\frac{\pi x^2}{4}+\frac{\pi x^2}{8}=100x-\frac{x^2}{2}-\frac{\pi x^2}{8}$$
$$S'=100-x-\frac{\pi x}{4}$$

$$100-x-\frac{\pi x}{4}=0$$
$$100=x\left(1+\frac{\pi}{4}\right)$$
$$100=x\cdot\frac{4+\pi}{4}$$
$$\boxed{\frac{400}{4+\pi}=x}$$



$$y=100-\frac{200}{4+\pi}-\frac{100\pi}{4+\pi}$$
$$=\frac{400+100\pi-200-100\pi}{4+\pi}=\underline{\underline{\frac{200}{4+\pi}}}$$

LETENKY

osob: I. $50\le x\le 100$ cena $=100$

II. $100< x\le 200$ cena $=100-0{,}5\cdot(x-100)=150-0{,}5x$

Max zisk I. zisk $=x\cdot$ cena $=x\cdot 100$ → tedy max zisk $=100\cdot 100=\underline{10000}$

II zisk $=x\cdot(150-0{,}5x)=150x-0{,}5x^2$

$$z'=150-x$$
$$150-x=0$$
$$150=x$$

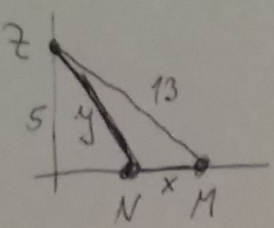
$\nearrow\ \searrow$ 150

max zisk $=150\cdot 150-0{,}5\cdot 150^2$
$=\underline{\underline{11.250}}$

Max. zisk bude, pokud poletí 150 osob.

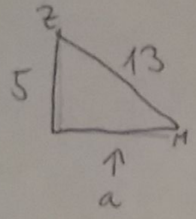
Scanned by CamScanner

CESTA



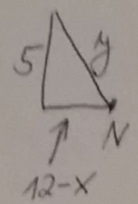
min Náklady

$N = 0{,}5x + 1{,}5y$



$13^2 = 5^2 + a^2$

$a = 12$



$y^2 = 5^2 + (12-x)^2$

$y^2 = 25 + 144 - 24x + x^2$

$y^2 = 169 - 24x + x^2$

$N = 1{,}5 \cdot \sqrt{169 - 24x + x^2} + 0{,}5x$

$N' = 1{,}5 \cdot \frac{1}{2\sqrt{169-24x+x^2}} \cdot (-24 + 2x) + 0{,}5$

$\frac{3(-24+2x)}{4\sqrt{169-24x+x^2}} + 0{,}5 = 0$

$3(-24+2x) + 2\sqrt{169-24x+x^2} = 0$

$\sqrt{169-24x+x^2} = (12-x) \cdot 3$

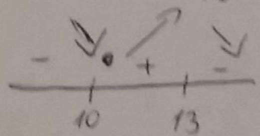
$(169 - 24x + x^2) = (144 - 24x + x^2) \cdot 9$

$-8x^2 + 192x - 1127 = 0$

$x_{1,2} = \frac{-192 \pm \sqrt{800}}{-16} = 12 \mp \frac{10\sqrt{8}}{16}$

$10{,}23$

$13{,}77$



$|NM| = 10{,}23$

PLECHOVKY

$V = 0{,}5\,l = 50\,cm^3$ $\qquad V = \pi r^2 \cdot v$

min S $\qquad S = 2\pi r^2 + 2\pi r v \qquad \frac{50}{\pi r^2} = v$

$$S = 2\pi r^2 + \frac{2\pi r \cdot 50}{\pi r^2} = 2\pi r^2 + \frac{100}{r}$$

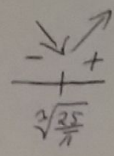
$$S' = 4\pi r - 100 r^{-2} = 4\pi r - \frac{100}{r^2}$$

$$4\pi r - \frac{100}{r^2} = 0$$

$$4\pi r^3 - 100 = 0$$

$$r^3 = \frac{25}{\pi}$$

$$v = \frac{50}{\pi \cdot \sqrt[3]{\frac{25^2}{\pi^2}}} = \underline{\underline{\sqrt[3]{\frac{200}{\pi}}}} \qquad \boxed{r = \sqrt[3]{\frac{25}{\pi}}}$$



VSTUPENKY

max zisk $\qquad$ ~~[illegible]~~

~~[illegible]~~

cena $40+x$

kusů $800-10x$

$$z = (40+x)\cdot(800-10x) - 20000$$

$$z = 32000 - 400x + 800x - 10x^2 - 20000$$

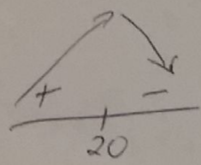
$$z = 12000 + 400x - 10x^2$$

$$z' = 400 - 20x$$

$$400 - 20x = 0$$

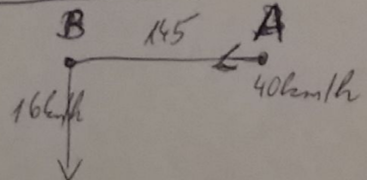
$$400 = 20x$$

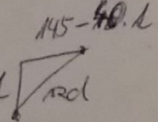
$$20 = x$$



Cena lístku 60\$.

PŘÍSTAVY





min vzd.

$$vzd^2 = (16t)^2 + (145-40t)^2$$

$$vzd = \sqrt{256t^2 + (145-40t)^2}$$

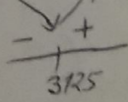
$$vzd' = \frac{512t + 2(145-40t)\cdot(-40)}{2\sqrt{256t^2 + (145-40t)^2}}$$

$$512t - 80(145-40t) = 0$$

$$512t - 11600 + 3200t = 0$$

$$3712t = 11600$$

$$\boxed{t = 3{,}125}$$



Jejich vzájemná vzdálenost bude nejmenší po 3 hod 7 min 30 s.

EMISE

$$\lim_{p \to 100^-} \frac{75000p}{100-p} = \left[\frac{7500000}{0^+}\right] = +\infty$$

Náklady na snižování emisí rostou neomezeně.

ZNEČIŠTĚNÍ JEZERA

$$f' = \frac{(4t-1)(2t^2+8)-(2t^2-t+8)\cdot 4t}{(2t^2+8)^2} =$$

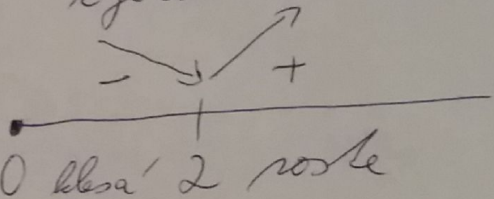
$$= \frac{8t^3+32t-2t^2-8-8t^3+4t^2-32t}{(2t^2+8)^2} = \frac{2t^2-8}{(2t^2+8)^2}$$

$$\frac{2t^2-8}{(2t^2+8)^2} = 0$$

$$2t^2 = 8$$

$$t = \pm 2$$

t je čas $\Rightarrow t \geq 0$



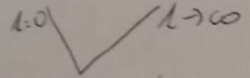
a) klesá 2 týdny

b) $t=2$

nejnižší je po 2 týdnech

c) $f(2) = \frac{2\cdot 4-2+8}{2\cdot 4+8} = \frac{14}{16} = \frac{7}{8}$

d) funkce klesá, pak roste



takže max je buď v $t=0$ nebo $t=\infty$

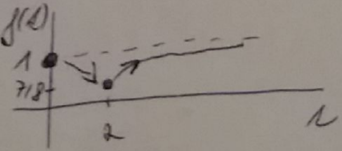
$t=0 \quad f(0)=1$

$t=\infty \quad \lim_{t\to\infty} f(t) = \lim_{t\to\infty} \frac{2t^2-t+8}{2t^2+8} = 1$ ← blíží se k 1

nejvyšší hladina kyslíku je v $t=0$.

e) nejvyšší hl. kyslíku je $f(0)=1$.

f) funkce vypadá takto:



Původní hodnoty nedosáhne, bude se tam jen blížit. Je tam asymptota. $y=1$

g) $\lim_{t\to\infty} f(t) = 1$

POPTÁVKA

a) $EP(c) = -\dfrac{c \cdot P'(c)}{P(c)}$

$P(c) = 200 \cdot (15 - 0{,}001c)^2$

$P'(c) = 200 \cdot 2 \cdot (15 - 0{,}001c) \cdot (-0{,}001)$

$c = 12\,000$

$EP(12000) = -\dfrac{12000 \cdot 200 \cdot 2 \cdot (15-12) \cdot (-0{,}001)}{200(15-12)^2}$

$EP(12000) = 8$ $\Rightarrow$ Poptávka je elastická ($EP > 1$), tedy zvýšení ceny způsobí pokles tržeb.

b) $EP(c) = \dfrac{\%\text{ změna popt}}{\%\text{ změna ceny}}$

$EP(c) \cdot \%ZC = \%ZP$

$8 \cdot 0{,}05 = \%ZP$

$0{,}4 = \%ZP$

Poptávka klesne o 40%.

c) $EP(c) = \dfrac{\%ZP}{\%ZC}$

$8 = \dfrac{0{,}05}{\%ZC}$

$\%ZC = \dfrac{0{,}05}{8} = 0{,}00625$

$0{,}625\%$

musíme snížit cenu o 0,625%.

VÝROBKY

a) rychlost = S'

$S'(t) = -5000\, e^{-0{,}25t} \cdot (-0{,}25)$

$S'(1) = -5000\, e^{-0{,}25} \cdot (-0{,}25) = 1250\, e^{-0{,}25} \doteq 973{,}5$

Rychlost je 973,5 jednotek za rok.

b) $\lim_{t \to \infty} 20000 - 5000\, e^{-0{,}25t} = 20\,000 - 5000 \cdot 0 = 20\,000$

Objem prodeje se ustálí na 20.000 jednotek.

ÚROČENÍ

spojité úročení $K_n = K_0 \cdot e^{i \cdot n}$ $K_0 = 1000$
$i = 0{,}06$

$K_n = 1000 \cdot e^{0{,}06n}$

a) $K'_n = 1000 \cdot e^{0{,}06n} \cdot 0{,}06 = 60\, e^{0{,}06n}$

$K'_5 = 60 \cdot e^{0{,}06 \cdot 5} = 80{,}99$

Na konci 5. tého roku bude hotovost narůstat rychlostí 80,99 Eur/rok. (neboli $\frac{80{,}99}{365} \doteq 0{,}222$ €/den)

b) $K_5 = 1000 \cdot e^{0{,}06 \cdot 5} = 1349{,}9$

Zůstatek bude 1349,9 Eur.