Pomocí diferenciálu určete přibližnou hodnotu:

1) $1{,}07^{10}$ $[1{,}7]$

2) $\sqrt{99{,}97}$ $[9{,}9985]$

3) $\ln 0{,}994$ $[-0{,}006]$

4) $\sqrt[3]{8{,}003}$ $[2{,}00025]$

5) $\dfrac{2{,}94^2 - 3\cdot 2{,}94}{2{,}94^2+3}$ $[-0{,}015]$

6) $\cancel{\cos 61^\circ}$ $\cancel{\left[\frac{1}{2} - \frac{\pi\sqrt{3}}{360}\right]}$

7) $e^{1{,}2}$ $[1{,}2e]$

8) $\operatorname{arctg} 1{,}01$ $[\frac{\pi}{4} + 0{,}005]$

9) $1{,}03^3$ $[1{,}09]$

Řešení

1)
$$f(x) = x^{10}$$
$$x = 1{,}07$$
$$x_0 = 1$$
$$f(x_0) = 1$$
$$f'(x) = 10x^9$$
$$f'(x_0) = 10$$
$$f(x) = f(x_0) + f'(x_0)\cdot(x - x_0)$$
$$f(x) = 1 + 10\cdot(1{,}07 - 1)$$
$$\underline{\underline{f(x) = 1{,}7}}$$

5)
$$f(x) = \frac{x^2 - 3\cdot x}{x^2+3}$$
$$x = 2{,}94$$
$$x_0 = 3$$
$$f(x_0) = \frac{9 - 3\cdot 3}{9+3} = 0$$
$$f'(x) = \frac{(2x-3)(x^2+3) - (x^2-3x)\cdot 2x}{(x^2+3)^2}$$
$$f'(x_0) = \frac{3\cdot 12 - 0}{12^2} = \frac{1}{4}$$
$$f(x) = 0 + \frac{1}{4}(2{,}94 - 3)$$
$$\underline{\underline{f(x) = -0{,}015}}$$

8)
$$f(x) = \operatorname{arctg} x$$
$$x = 1{,}01$$
$$x_0 = 1$$
$$f(x_0) = \frac{\pi}{4}$$
$$f'(x) = \frac{1}{1+x^2}$$
$$f'(x_0) = \frac{1}{2}$$
$$f(x) = \frac{\pi}{4} + \frac{1}{2}(1{,}01 - 1)$$
$$\underline{\underline{f(x) = \frac{\pi}{4} + 0{,}005}}$$