Určete rovnici tečny a normály:

a) $f(x) = \ln(x+1)$ $\quad T[0; ?]$ $\quad t: y = x$ $\quad n: y = -x$

b) $f(x) = \sin(2x)$ $\quad T[\frac{3}{4}\pi; ?]$ $\quad t: y = -1$ $\quad n:$ neexistuje

c) $f(x) = \dfrac{1+x^3}{x-1}$ $\quad T[2; ?]$ $\quad t: y = 3x+3$ $\quad n: y = \frac{-x}{3} + \frac{29}{9}$

d) $f(x) = (2x-1)\cdot e^{3x}$ $\quad T[0; ?]$ $\quad t: y+1 = -x$ $\quad n: y+1 = x$

f) $f(x) = x^3 \cdot \ln x$ $\quad T[1; ?]$ $\quad t: y = x-1$ $\quad n: y = -x+1$

g) $f(x) = \dfrac{2x-1}{3x+2}$ $\quad T[-1; ?]$ $\quad t: y-3 = 7(x+1)$ $\quad n: y-3 = -\frac{1}{7}(x+1)$

h) $f(x) = e^{3x-2}$ $\quad T[1; ?]$ $\quad t: y-e = 3e(x-1)$ $\quad n: y-e = -\frac{1}{3e}(x-1)$

i) $f(x) = (x+3)e^{3x-1}$ $\quad T[\frac{1}{3}; ?]$ $\quad t: y-\frac{10}{3} = 11(x-\frac{1}{3})$ $\quad n: y-\frac{10}{3} = -\frac{1}{11}(x-\frac{1}{3})$

j) $f(x) = (5x+3-x^2)\cdot e^{4-x}$ $\quad T[4; ?]$ $\quad t: y-7 = 4(x-4)$ $\quad n: y-7 = -\frac{1}{4}(x-4)$

ŘEŠENÝ PŘÍKLAD:

I. $f(x) = (3x+5)\cdot e^{-3x}$ $\quad T[0; ?]$

$$y_0 = (3\cdot 0+5)\cdot e^{-3\cdot 0} = 5\cdot 1 = 5$$

$$f'(x) = 3\cdot e^{-3x} + (3x+5)\cdot e^{-3x}\cdot(-3)$$

$$f'(x_0) = 3\cdot e^{-3\cdot 0} + (3\cdot 0+5)\cdot e^{-3\cdot 0}\cdot(-3) = 3\cdot 1 + 5\cdot 1\cdot(-3) = -12$$

$t: y-5 = -12(x-0)$ $\qquad n: y-5 = -\frac{1}{-12}(x-0)$

$\quad y-5 = -12x$ $\qquad\qquad y-5 = \frac{1}{12}x$

II. $f(x) = \sin x - \cos 2x$ $\quad T[\frac{\pi}{2}; ?]$

$$y_0 = \sin\frac{\pi}{2} - \cos 2\cdot\frac{\pi}{2} = 1-(-1) = 2$$

$$f'(x) = \cos x + (\sin 2x)\cdot 2$$

$$f'(x_0) = \cos\frac{\pi}{2} + (\sin\pi)\cdot 2 = 0+0 = 0$$

$t: y-2 = 0\cdot(x-\frac{\pi}{2})$ $\qquad n:$ neexistuje $\quad y-2 = -\frac{1}{0}(x-\frac{\pi}{2})$

$\quad y-2 = 0$ $\qquad$ (protože nelze dosadit)

Najděte rovnici tečny a normály ke grafu funkce $y = f(x)$ v bodě $x = a$

(a) $y = \sqrt{x}, \;\; a = 9$

(b) $y = x^3 + 2x, \;\; a = 1$

(c) $y = \dfrac{3x-4}{2x-3}, \;\; a = 2$

(d) $y = 2\sqrt{2}\sin x, \;\; a = \dfrac{\pi}{4}$

(e) $y = \ln(x+1), \;\; a = 0$

(f) $y = \mathrm{e}^{-x}\cos 2x, \;\; a = 0$

(~~kombinované studium pouze tečnu~~)

Řešení:

(a) $t : x - 6y + 9 = 0, n : 6x + y - 57 = 0$; (b) $t : 5x - y - 2 = 0, n : x + 5y - 16 = 0$;
(c) $t : x + y - 4 = 0, n : x - y = 0$; (d) $t : 2x - y + 2 - \frac{\pi}{2} = 0, n : x + 2y - 4 - \frac{\pi}{4} = 0$;
(e) $t : y = x, n : y = -x$; (f) $t : x + y - 1 = 0, n : x - y + 1 = 0$