Určete rovnici tečny a normály:

a) $f(x)=\ln(x+1)$ $T[0;?]$ $t: y=x$ $n: y=-x$

b) $f(x)=\sin(2x)$ $T[\frac{3}{4}\pi;?]$ $t: y=-1$ $n:$ neexistuje

c) $f(x)=\frac{1+x^3}{x-1}$ $T[2;?]$ $t: y=3x+3$ $n: y=\frac{-x}{3}+\frac{29}{9}$

d) $f(x)=(2x-1)\cdot e^{3x}$ $T[0;?]$ $t: y+1=-x$ $n: y+1=x$

f) $f(x)=x^3\cdot\ln x$ $T[1;?]$ $t: y=x-1$ $n: y=-x+1$

g) $f(x)=\frac{2x-1}{3x+2}$ $T[-1;?]$ $t: y-3=7(x+1)$ $n: y-3=-\frac{1}{7}(x+1)$

h) $f(x)=e^{3x-2}$ $T[1;?]$ $t: y-e=3e(x-1)$ $n: y-e=-\frac{1}{3e}(x-1)$

i) $f(x)=(x+3)e^{3x-1}$ $T[\frac{1}{3};?]$ $t: y-\frac{10}{3}=11(x-\frac{1}{3})$ $n: y-\frac{10}{3}=\frac{-1}{11}(x-\frac{1}{3})$

j) $f(x)=(5x+3-x^2)\cdot e^{4-x}$ $T[4;?]$ $t: y-7=4(x-4)$ $n: y-7=-\frac{1}{4}(x-4)$

ŘEŠENÝ PŘÍKLAD:

I. $f(x)=(3x+5)\cdot e^{-3x}$ $T[0;?]$

$$y_0=(3\cdot 0+5)\cdot e^{-3\cdot 0}=5\cdot 1=5$$

$$f'(x)=3\cdot e^{-3x}+(3x+5)\cdot e^{-3x}\cdot(-3)$$

$$f'(x_0)=3\cdot e^{-3\cdot 0}+(3\cdot 0+5)\cdot e^{-3\cdot 0}\cdot(-3)=3\cdot 1+5\cdot 1\cdot(-3)=-12$$

$t: y-5=-12(x-0)$ $n: y-5=-\frac{1}{-12}(x-0)$

$y-5=-12x$ $y-5=\frac{1}{12}x$

II. $f(x)=\sin x-\cos 2x$ $T[\frac{\pi}{2};?]$

$$y_0=\sin\frac{\pi}{2}-\cos 2\cdot\frac{\pi}{2}=1-(-1)=2$$

$$f'(x)=\cos x+(\sin 2x)\cdot 2$$

$$f'(x_0)=\cos\frac{\pi}{2}+(\sin\pi)\cdot 2=0+0=0$$

$t: y-2=0\cdot(x-\frac{\pi}{2})$ $n:$ neexistuje $y-2=-\frac{1}{0}(x-\frac{\pi}{2})$

$y-2=0$ (protože nelze dosadit)

Najděte rovnici tečny a normály ke grafu funkce $y = f(x)$ v bodě $x = a$

(a) $y = \sqrt{x},\ a = 9$

(b) $y = x^3 + 2x,\ a = 1$

(c) $y = \dfrac{3x-4}{2x-3},\ a = 2$

(d) $y = 2\sqrt{2}\sin x,\ a = \dfrac{\pi}{4}$

(e) $y = \ln(x+1),\ a = 0$

(f) $y = \mathrm{e}^{-x}\cos 2x,\ a = 0$

(~~kombinované studium pouze tečnu~~)

Řešení:

(a) $t: x - 6y + 9 = 0, n: 6x + y - 57 = 0$; (b) $t: 5x - y - 2 = 0, n: x + 5y - 16 = 0$;
(c) $t: x + y - 4 = 0, n: x - y = 0$; (d) $t: 2x - y + 2 - \frac{\pi}{2} = 0, n: x + 2y - 4 - \frac{\pi}{4} = 0$;
(e) $t: y = x, n: y = -x$; (f) $t: x + y - 1 = 0, n: x - y + 1 = 0$

Určete parametr q tak, aby přímka byla tečnou grafu fce $f(x)$:

a) $p: x - y + q = 0$ $\quad f(x): y = -x^2 + 11x + 8$ $\quad [q = 33]$

b) $p: 4x - 2y + q = 0$ $\quad f(x): y = -4x^2 + 10x - 2$ $\quad [q = 4]$

c) $p: 2x - y + q = 0$ $\quad f(x): y = -x^2 + 10x + 3$ $\quad [q = 19]$

d) $p: 3x - 2y + q = 0$ $\quad f(x): y = -2x^2 + 3x - 1$ $\quad [q = -\frac{23}{16}]$

e) $p: -2x + y + q = 0$ $\quad f(x): y = x^2 + 2x + 2$ $\quad [q = -2]$

Určete parametr q tak, aby přímka byla normálou grafu fce $f(x)$:

a) $p: -3x - 15y + q = 0$ $\quad y = 2x^2 - 11x + 8$ $\quad [-48]$

b) $p: -x + 4y + q = 0$ $\quad y = x^2 - 8x + 9$ $\quad [14]$

c) $p: -2x + y + q = 0$ $\quad y = -x^2 + 3x + 2$ $\quad [q = -\frac{11}{16}]$

d) $p: 2x - 3y + q = 0$ $\quad y = -2x^2 + 5x - 1$ $\quad [q = \frac{73}{32}]$

e) $p: 3x + 2y + q = 0$ $\quad y = -x^2 + x - 2$ $\quad [q = \frac{29}{9}]$

f) $p: x + 5y + q = 0$ $\quad y = x^2 + 3x - 6$ $\quad [q = -26]$

g) $p: x - 5y + q = 0$ $\quad y = -2x^2 + 3x + 1$ $\quad [q = -7]$