Vypočítejte integrály:

$$\int e^{3x} - \cos x + \frac{2}{x} + \frac{3}{2-x} - \frac{\cos x}{\sin x - 4} dx$$

$$\int \sqrt[3]{x^2} - \frac{1}{\sqrt{x}} + \frac{\sqrt[3]{x}}{\sqrt{x}} + 2dx$$

$$\int \frac{2x-5}{x^2} + (x-4)^2 + 3dx$$

$$\int \frac{2}{1+x^2} - \frac{x}{x^2+1} + \frac{3}{2x-1} dx$$

$$\int 2\cos x - 3\sin x + \frac{2-\sin^3 x}{\sin^2 x} dx$$

$$\int e^{-x} + 3x\sqrt{x} - \frac{1}{x^2} - 1dx$$

$$\int \frac{2x+3}{x^2+3x+1} - \frac{2x}{x^2-4} + \frac{2}{\sqrt{1-x^2}} + 3^x dx$$

$$\int \frac{1}{x+1} - \frac{1}{x^3} + \frac{1}{2-x} + \frac{x}{x^2-5} dx$$

$$\int x \sin x dx$$

$$\int x^2 e^{-x} dx$$

$$\int x^2 \cos x dx$$

$$\int x \cdot arctgx dx$$

$$\int x^3 \ln x dx$$

$$\int x \cdot 2^x dx$$

$$\int \cos^2 x dx$$

Vypočítejte integrály:

- $\int e^{3x} - \cos x + \frac{2}{x} + \frac{3}{2-x} - \frac{\cos x}{\sin x - 4}\,dx = \frac{e^{3x}}{3} - \sin x + 2\ln|x| - 3\ln|2-x| - \ln|\sin x - 4| + c$

- $\int \sqrt[3]{x^2} - \frac{1}{\sqrt{x}} + \frac{\sqrt[3]{x}}{\sqrt{x}} + 2\,dx = \int x^{\frac{2}{3}} - x^{-\frac{1}{2}} + x^{\frac{-1}{6}} + 2\,dx =$ $\left(= x^{\frac{1}{3}-\frac{1}{2}}\right)$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{6}}}{\frac{5}{6}} + 2x = \frac{3\sqrt[3]{x^5}}{5} - 2\sqrt{x} + \frac{6\sqrt[6]{x^5}}{5} + 2x + c$$

- $\int \frac{2x-5}{x^2} + (x-4)^2 + 3\,dx = \int \frac{2}{x} - \frac{5}{x^2} + x^2 - 8x + 16 + 3\,dx =$

$$= \int \frac{2}{x} - 5x^{-2} + x^2 - 8x + 19\,dx = 2\ln|x| - \frac{5x^{-1}}{-1} + \frac{x^3}{3} - \frac{8x^2}{2} + 19x = 2\ln|x| + \frac{5}{x} + \frac{x^3}{3} - 4x^2 + 19x + c$$

- $\int \frac{2}{1+x^2} - \frac{x}{x^2+1} + \frac{3}{2x-1}\,dx = \int 2\cdot\frac{1}{1+x^2} - \frac{1}{2}\cdot\frac{2x}{x^2+1} + \frac{3}{2x-1}\,dx =$

$$= 2\,\mathrm{arctg}\,x - \frac{1}{2}\ln|x^2+1| + \frac{3\ln|2x-1|}{2} + c$$

- $\int 2\cos x - 3\sin x + \frac{2-\sin^3 x}{\sin^2 x}\,dx = \int 2\cos x - 3\sin x + \frac{2}{\sin^2 x} - \sin x\,dx$

$$= 2\sin x + 3\cos x - 2\,\mathrm{cotg}\,x + \cos x + c$$

- $\int e^{-x} + 3x\cdot\sqrt{x} - \frac{1}{x^2} - 1\,dx = \int e^{-x} + 3x^{\frac{3}{2}} - x^{-2} - 1\,dx =$

$$= -e^{-x} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{e^{-1}}{-1} - x = -e^{-x} + \frac{6\sqrt{x^5}}{5} + \frac{1}{e} - x + c$$

- $\int \frac{2x+3}{x^2+3x+1} - \frac{2x}{x^2-4} + \frac{2}{\sqrt{1-x^2}} + 3^x\,dx = \ln|x^2+3x+1| - \ln|x^2-4| + 2\arcsin x + \frac{3^x}{\ln 3} + c$

- $\int \frac{1}{x+1} - \frac{1}{x^3} + \frac{1}{2-x} + \frac{x}{x^2-5}\,dx = \int \frac{1}{x+1} - x^{-3} + \frac{1}{2-x} + \frac{1}{2}\cdot\frac{2x}{x^2-5}\,dx =$

$$= \ln|x+1| - \frac{x^{-2}}{-2} + \frac{\ln|2-x|}{-1} + \frac{1}{2}\ln|x^2-5| = \ln|x+1| + \frac{1}{2x^2} - \ln|2-x| + \frac{1}{2}\ln|x^2-5| + c$$

- $\int x\cdot \sin x\, dx = \left| \begin{array}{ll} f'=\sin x & f=-\cos x \\ g=x & g'=1 \end{array} \right| = -x\cos x - \int -\cos x\, dx = -x\cos x + \sin x + c$

- $\int x^2\cdot e^{-x} dx = \left| \begin{array}{ll} f'=e^{-x} & f=-e^{-x} \\ g=x^2 & g'=2x \end{array} \right| = -x^2\cdot e^{-x} - \int 2x\cdot(-e^{-x})\, dx = -x^2\cdot e^{-x} +$

$+\int 2x\cdot e^{-x} dx = \left| \begin{array}{ll} f'=e^{-x} & f=-e^{-x} \\ g=2x & g'=2 \end{array} \right| =$

$= -x^2\cdot e^{-x} - 2x e^{-x} - \int -2e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$

- $\int x^2\cdot \cos x\, dx = \left| \begin{array}{ll} f'=\cos x & f=\sin x \\ g=x^2 & g'=2x \end{array} \right| = x^2\cdot \sin x - \int 2x \sin x\, dx =$

$\left| \begin{array}{ll} f'=\sin x & f=-\cos x \\ g=2x & g'=2 \end{array} \right| = x^2\cdot \sin x - \left(-2x\cos x - \int -2\cos x\, dx\right)$

$= x^2\cdot \sin x + 2x\cos x - 2\sin x + c$

- $\int x\cdot \operatorname{arctg} x\, dx = \left| \begin{array}{ll} f'=x & f=\frac{x^2}{2} \\ g=\operatorname{arctg} x & g'=\frac{1}{1+x^2} \end{array} \right| = \frac{x^2}{2}\operatorname{arctg} x -$

$\int \frac{x^2}{2}\cdot\frac{1}{x^2+1}\, dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2}{x^2+1}\, dx =$

$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\, dx = \frac{x^2}{2}\operatorname{arctg} x -$

$-\frac{1}{2}\int 1 - \frac{1}{x^2+1}\, dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + c$

- $\int x^3\cdot \ln x\, dx = \left| \begin{array}{ll} f'=x^3 & f=\frac{x^4}{4} \\ g=\ln x & g'=\frac{1}{x} \end{array} \right| = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\cdot\frac{1}{x}\, dx = \frac{x^4}{4}\ln x -$

$-\int \frac{1}{4}\cdot x^3 dx = \frac{x^4}{4}\ln x - \frac{1}{4}\frac{x^4}{4} = \frac{x^4}{4}\ln x - \frac{x^4}{16} + c$

- $\int x\cdot 2^x dx = \left| \begin{array}{ll} f'=2^x & f=\frac{2^x}{\ln 2} \\ g=x & g'=1 \end{array} \right| = \frac{x\cdot 2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\, dx = \frac{x\cdot 2^x}{\ln 2} - \frac{1}{\ln 2}\cdot\int 2^x dx =$

$= \frac{x\cdot 2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2} + c$

→ môže $\sin^2 x = 1-\cos^2 x$

- $\int \cos^2 x\, dx = \left| \begin{array}{ll} f'=\cos x & f=\sin x \\ g=\cos x & g'=-\sin x \end{array} \right| = \sin x\cdot\cos x - \int -\sin^2 x = \sin x\cos x + \int 1-\cos^2 x\, dx =$

$= \sin x\cos x + x - \int \cos^2 x\, dx$

dopočítáme jako rovnici

$\int \cos^2 x\, dx = \sin x\cos x + x - \int \cos^2 x\, dx$

$2\int \cos^2 x\, dx = \sin x\cos x + x$

$\int \cos^2 x\, dx = \frac{\sin x\cos x + x}{2} + c$