

1) Urcete definicní obor funkce:

$$f(x) = \sqrt{x^2 - x - 2} + \ln \frac{x-3}{-2-x}$$

2) Vypocítejte limitu

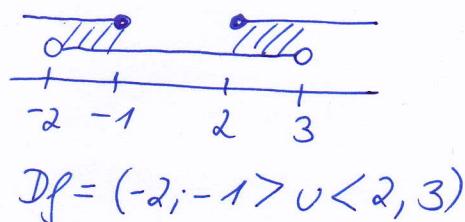
$$\lim_{x \rightarrow \infty} x^2 e^{-x}$$

3) Urcete intervaly, na kterých je funkce $f(x) = 4x^3 - 3x^4$ rostoucí ci klesající, najdete lokální extrémy.

4) Najděte všechny asymptoty funkce $f(x) = \frac{2x^2 - x}{3 - x}$.

$$\begin{aligned} \textcircled{1} \quad & x^2 - x - 2 \geq 0 \\ & (x-2)(x+1) \geq 0 \\ & \begin{array}{c} + \\ - \\ + \end{array} \end{aligned}$$

$$\begin{aligned} & \frac{x-3}{-2-x} > 0 \\ & \begin{array}{c} - \\ + \\ - \end{array} \end{aligned}$$



$$\textcircled{2} \quad \lim_{x \rightarrow \infty} x^2 \cdot e^{-x} = [\infty, 0] = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{2}{e^x} = \left[\frac{2}{\infty} \right] = 0$$

$$\textcircled{3} \quad f' = 12x^2 - 12x^3 \quad Df = \mathbb{R}$$

$$12x^2 - 12x^3 = 0$$

$$12x^2(1-x) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ x=0 & & x=1 \\ + & + & - \end{array}$$

rost. $(-\infty, 0), (0, 1)$

kles. $(1, \infty)$

lok. max. $[1; 1]$

lok. min. nem'

$$\textcircled{4} \quad Df = \mathbb{R} \setminus \{3\}$$

$$\lim_{x \rightarrow 3^+} \frac{2x^2 - x}{3 - x} = \left[\frac{15}{0^-} \right] = -\infty \Rightarrow \text{as. bez směrnice } \underline{\underline{x=3}}$$

$$k = \lim_{x \rightarrow \pm\infty} \frac{2x^2 - x}{3 - x} = \lim_{x \rightarrow \pm\infty} \frac{2x^2 - x}{3x - x^2} = -2$$

$$g = \lim_{x \rightarrow \pm\infty} \frac{2x^2 - x}{3 - x} + 2x = \lim_{x \rightarrow \pm\infty} \frac{2x^2 - x + 6x - 2x^2}{3 - x} = \lim_{x \rightarrow \pm\infty} \frac{5x}{3 - x} = -5$$

$$\text{as. se směrem. } \underline{\underline{y = -2x - 5}}$$

1) Urcete definicní obor funkce:

$$f(x) = \sqrt{\frac{x+1}{5+x}} + \ln(-x^2 - x + 12)$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow 0} \frac{2 \operatorname{arctg} 3x}{5x}$$

3) Urcete intervaly, na kterých je funkce $f(x) = x \ln x$ konvexní ci konkávní, najdete inflexní body.

4) Urcete rovnici tecny a normály ke grafu funkce $f(x) = \frac{3x-4}{2x-3}$ v bode T = [2; ?].

① $\frac{x+1}{5+x} \geq 0 \quad x \neq -5 \quad -x^2 - x + 12 > 0$

$\begin{array}{c} + \\ \textcircled{+} \\ - \\ -5 \end{array} \quad \begin{array}{c} - \\ \textcircled{+} \\ -1 \end{array}$

$\begin{array}{c} - \\ \textcircled{-} \\ 0 \end{array} \quad \begin{array}{c} + \\ \textcircled{+} \\ - \\ -4 \end{array} \quad \begin{array}{c} - \\ \textcircled{-} \\ 3 \end{array}$

$Df = (-1, 3)$

② $\lim_{x \rightarrow 0} \frac{2 \operatorname{arctg} 3x}{5x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2 \cdot \frac{1}{1+(3x)^2} \cdot 3}{5} = \frac{2 \cdot 1 \cdot 3}{5} = \underline{\underline{\frac{6}{5}}}$

③ $f' = \ln x + x \cdot \frac{1}{x} = \ln x + 1$
 $f'' = \frac{1}{x}$
 $\frac{1}{x} = 0 \quad 1 \neq 0 \text{ nem' inf. bod}$

$Df = (0, \infty)$

$f \text{ je konkávní } (0, +\infty)$
 (na celém Df)

④ $x_0 = 2 \quad y_0 = \frac{3 \cdot 2 - 4}{2 \cdot 2 - 3} = 2$

$$f' = \frac{3(2x-3) - (3x-4) \cdot 2}{(2x-3)^2} = \frac{6x-9-6x+8}{(2x-3)^2} = \frac{-1}{(2x-3)^2}$$

$$f'(2) = \frac{-1}{(2 \cdot 2 - 3)^2} = -1$$

l: $y = -1(x-2) + 2$
 $\underline{y = -x + 4}$

m: $y = \frac{-1}{-1}(x-2) + 2$
 $\underline{y = x}$

1) Urcete definicní obor funkce:

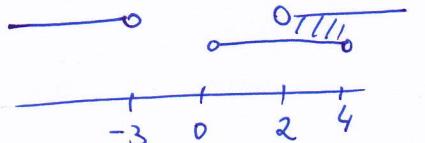
$$f(x) = \frac{\ln(x^2 + x - 6)}{\sqrt{4x - x^2}}$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow \infty} \frac{\log x}{1-x}$$

3) Urcete intervaly, na kterých je funkce $f(x) = e^{-\frac{1}{2}x^2}$ konvexní ci konkávní, najdete inflexní body.4) Najdete inverzní funkci k funkci $f(x) = 3^{x-4} + 2$. Urcete definicní obor a obor hodnot obou dvou funkcí.

$$\textcircled{1} \quad x^2 + x - 6 > 0 \quad 4x - x^2 > 0$$



$$(x+3)(x-2) > 0 \quad x(4-x) > 0$$

$$\begin{array}{c} \textcircled{+} \\ \hline -3 \end{array} \quad \begin{array}{c} \textcircled{-} \quad \textcircled{+} \\ \hline 0 \quad 4 \end{array}$$

$$Df = (2, 4)$$

$$\textcircled{2} \quad \lim_{x \rightarrow \infty} \frac{\log x}{1-x} = \left[\frac{\infty}{-\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x \cdot \ln 10}}{-1} = \left[-\frac{1}{\infty} \right] = 0$$

$$\textcircled{3} \quad f' = e^{-\frac{1}{2}x^2} \cdot (-\frac{1}{2} \cdot 2x) = e^{-\frac{1}{2}x^2} \cdot (-x)$$

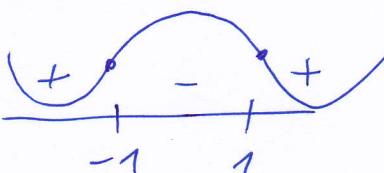
$$f'' = e^{-\frac{1}{2}x^2} \cdot (-x)^2 + e^{-\frac{1}{2}x^2} \cdot (-1) = e^{-\frac{1}{2}x^2}(x^2 - 1)$$

$$e^{-\frac{1}{2}x^2}(x^2 - 1) = 0$$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

$$Df = \mathbb{R}$$



konvexní $(-\infty, -1]$,
 $<1, +\infty)$

konkávní $(-1, 1)$

infl. body $[-1, e^{-\frac{1}{2}}]$
 $[1, e^{-\frac{1}{2}}]$

$$\textcircled{4} \quad f^{-1}: x = 3^{y-4} + 2 \quad Df = \mathbb{R} = Hf^{-1}$$

$$x - 2 = 3^{y-4}$$

$$\log_3(x-2) = y-4$$

$$\underline{\underline{\log_3(x-2) + 4 = y}}$$

$$\begin{aligned} \rightarrow x-2 &> 0 \\ x &> 2 \end{aligned}$$

$$Df^{-1} = (2, +\infty) = Hf$$

1) Určete definiční obor funkce:

$$f(x) = \ln \frac{2x+6}{4-x} - \sqrt{x^2 - 6x + 5}$$

2) Vypočítejte limitu

$$\lim_{x \rightarrow \infty} \frac{e^{4x}}{3x^2}$$

3) Určete intervaly, na kterých je funkce $f(x) = x + \frac{4}{x}$ rostoucí či klesající, najděte lokální extrémy.

4) Určete rovnici tečny ke grafu funkce $f(x) = \ln(2x+1)$ v bodě $T[0;?]$ a pomocí diferenciálu vypočítejte přibližnou hodnotu funkce $f(0,01)$.

$$\textcircled{1} \quad \frac{2x+6}{4-x} > 0$$

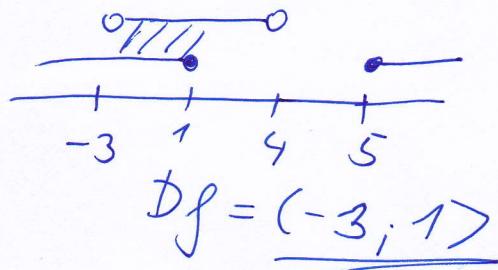
$\begin{array}{c} + \\ \hline - & + & - \end{array}$

$x = -3, x = 4$

$$\begin{aligned} x^2 - 6x + 5 &\geq 0 \\ (x-5)(x-1) &\geq 0 \end{aligned}$$

$\begin{array}{c} + \\ \hline - & + & + \end{array}$

$x = 1, x = 5$



$$\textcircled{2} \quad \lim_{x \rightarrow \infty} \frac{e^{4x}}{3x^2} = \left[\frac{\infty}{\infty} \right] \stackrel{CH}{=} \lim_{x \rightarrow \infty} \frac{e^{4x} \cdot 4}{6x} = \left[\frac{\infty}{\infty} \right] \stackrel{CH}{=} \lim_{x \rightarrow \infty} \frac{e^{4x} \cdot 16}{6} = \infty$$

$$\textcircled{3} \quad f' = 1 + 4 \cdot (-x^{-2}) = 1 - \frac{4}{x^2}$$

$$1 - \frac{4}{x^2} = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$\begin{array}{c} + & - & - & + \\ \hline -2 & 0 & 2 \end{array}$

$Df = \mathbb{R} \setminus \{0\}$

rost. $(-\infty, -2], [2, \infty)$
 kles. $(-2, 0), (0, 2)$
 lok. max. $[-2, -4]$
 lok. min. $[2, 4]$

$$\textcircled{4} \quad x_0 = 0 \quad y_0 = \ln(2 \cdot 0 + 1) = 0$$

$$f' = \frac{1}{2x+1} \cdot 2 = \frac{2}{2x+1} \quad f'(0) = \frac{2}{2 \cdot 0 + 1} = 2$$

$$t: \quad y = 2(x-0) + 0 \quad m: \quad y = \frac{1}{2}(x-0) + 0$$

$\begin{array}{c} 0 & + & - & + \\ \hline & & 1 & 2 \\ & & x & \end{array}$

$$f(0,01) = 2 \cdot 0,01 = \underline{0,02}$$

1) Urcete definicní obor funkce:

$$f(x) = \frac{\ln(x^2 + 6x + 8)}{x - 3} - \sqrt{2x - 1}$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow 0} \frac{\arcsin x}{1 - e^x}$$

3) Urcete intervaly, na kterých je funkce $f(x) = x^2 e^{-x}$ rostoucí ci klesající, najdete lokální extrémy.

4) Najdete rovnici inverzní funkce k funkci $f(x) = \log_2(x+3) - 4$. Urcete definicní obor a obor hodnot obou funkcí.

① $x^2 + 6x + 8 > 0$

$$(x+4)(x+2) > 0$$

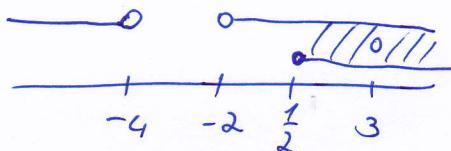
$$\begin{array}{c} + \\ - \\ + \end{array}$$

$$x - 3 \neq 0$$

$$x \neq 3$$

$$2x - 1 \geq 0$$

$$x \geq \frac{1}{2}$$



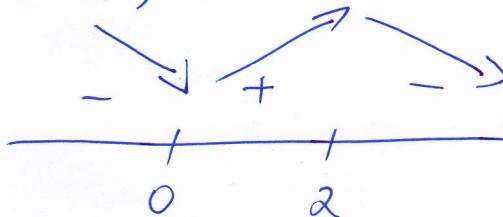
$$Df = \left\langle \frac{1}{2}; \infty \right\rangle - \{3\}$$

② $\lim_{x \rightarrow 0} \frac{\arcsin x}{1 - e^x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{-e^x} = \frac{1}{-1} = -1$

③ $f' = 2x \cdot e^{-x} + x^2 \cdot e^{-x}(-1) = e^{-x}(2x - x^2)$

$$Df = R$$

$$\begin{aligned} e^{-x}(2x - x^2) &= 0 \\ 2x - x^2 &= 0 \\ x(2 - x) &= 0 \\ x = 0 & \quad x = 2 \end{aligned}$$



kles. $(-\infty, 0)$, $(2, \infty)$

rost. $(0, 2)$

lok. min. $[0, 0]$

lok. max. $[2, 4e^{-2}]$

④ $f^{-1}: y = \log_2(y+3) - 4$

$$x+4 = \log_2(y+3)$$

$$2^{x+4} = y+3$$

$$\underline{2^{x+4} - 3 = y}$$

$$\begin{array}{l} x+3 > 0 \\ x > -3 \end{array} \quad Df = (-3, \infty) = Hf^{-1}$$

$$\rightarrow Df^{-1} = R = Hf$$

1) Určete definiční obor funkce:

$$f(x) = \frac{\ln(-x^2 + x + 12)}{\sqrt{2x^2 - 2}}$$

2) Vypočítejte limitu

$$\lim_{x \rightarrow 0} \frac{x \cos x}{2 \sin x}$$

3) Určete intervaly, na kterých je funkce $f(x) = x + \frac{1}{x^2}$ konvexní či konkávní, najděte inflexní body.

4) Určete rovnici tečny a normály ke grafu funkce $f(x) = 3x^2 - 2\sqrt{x}$ v bodě T[1,?].

① $-x^2 + x + 12 > 0$

$$-(x^2 - x - 12) > 0$$

$$-(x-4)(x+3) > 0$$

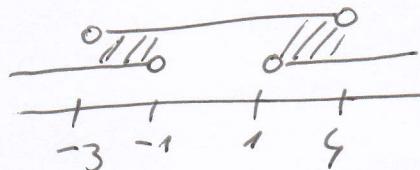
$$\begin{array}{c|ccc|c} & - & + & - & \\ \hline -3 & & & & 4 \end{array}$$

$$2x^2 - 2 > 0$$

$$2(x^2 - 1) > 0$$

$$2(x-1)(x+1) > 0$$

$$\begin{array}{c|ccc|c} & + & - & + & \\ \hline -1 & & & & 1 \end{array}$$



$$Df = (-3, -1) \cup (1, 4)$$

② $\lim_{x \rightarrow 0} \frac{x \cos x}{2 \sin x} = \left[\frac{0}{0} \right] \stackrel{\text{l'H}}{=} \lim_{x \rightarrow 0} \frac{\cos x + x(-\sin x)}{2 \cos x} = \frac{1+0}{2} = \underline{\underline{\frac{1}{2}}}$

③ $f(x) = 1 + (-2x^{-3}) = 1 - 2x^{-3}$

$$f'' = -2 \cdot (-3x^{-4}) = 6x^{-4} = \frac{6}{x^4} \quad \frac{6}{x^4} = 0$$

$$Df = R \setminus \{0\} \quad \begin{array}{c|cc|c} & + & + & \\ \hline 0 & & & \end{array} \quad 6 \neq 0 \text{ není infl. bod} \\ \text{konkav' } (-\infty, 0), (0, \infty)$$

④ $x_0 = 1 \quad y_0 = 3 \cdot 1 - 2\sqrt{1} = 1$

$$f' = 6x - 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} = 6x - \frac{1}{\sqrt{x}} \quad f'(1) = 6 - \frac{1}{1} = 5$$

$t: y = 5(x-1) + 1$

$$\underline{\underline{y = 5x - 4}}$$

$n: y = -\frac{1}{5}(x-1) + 1$

$$\underline{\underline{y = -\frac{1}{5}x + \frac{6}{5}}}$$

1) Určete definiční obor funkce:

$$f(x) = \ln \frac{x-2}{3+x} - \sqrt{x^2 - 4x + 3}$$

2) Vypočítejte limitu

$$\lim_{x \rightarrow 0} \frac{\arctg 2x}{2 \arcsin x}$$

3) Určete intervaly, na kterých je funkce $f(x) = \frac{e^x}{1+x}$ rostoucí či klesající a najděte lokální extrémy.4) Najděte rovnice všech asymptot funkce $f(x) = \frac{3x^2 - 1}{x + 1}$.

$$\textcircled{1} \quad \frac{x-2}{3+x} > 0$$

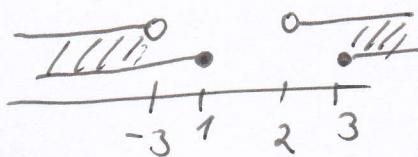
$\begin{array}{c} + \\ - \end{array}$

$\begin{array}{c} + \\ - \end{array}$

$$x^2 - 4x + 3 \geq 0$$

$$(x-3)(x-1) \geq 0$$

$$\begin{array}{c} + \\ - \end{array}$$



$$Df = (-\infty, -3) \cup (3, \infty)$$

$$\textcircled{2} \quad \lim_{x \rightarrow 0} \frac{\arctg 2x}{2 \arcsin x} = \left[\begin{matrix} 0 \\ 0 \end{matrix} \right] = \lim_{x \rightarrow 0} \frac{\frac{1}{1+4x^2} \cdot 2}{2 \cdot \frac{1}{\sqrt{1-x^2}}} = \frac{\frac{1}{1+0} \cdot 2}{2 \cdot \frac{1}{1}} = \frac{1}{2}$$

$$\textcircled{3} \quad f' = \frac{e^x \cdot (1+x) - e^x \cdot 1}{(1+x)^2} = \frac{e^x(1+x-1)}{(1+x)^2} = \frac{x \cdot e^x}{(1+x)^2}$$

$\begin{array}{c} - \\ + \end{array}$

$\begin{array}{c} - \\ + \end{array}$

$\begin{array}{c} - \\ + \end{array}$

kles. $(-\infty, -1), (-1, 0)$ rost. $(0, \infty)$ lok. min. $[0; 1]$

$$\textcircled{4} \quad Df = R \setminus \{-1\} \quad \lim_{x \rightarrow -1^+} \frac{3x^2 - 1}{x + 1} = \left[\frac{2}{0^+} \right] = +\infty \Rightarrow \text{as. bes. směrnice}$$

$x = -1$

$$k = \lim_{x \rightarrow \pm\infty} \frac{\frac{3x^2 - 1}{x}}{x} = \lim_{x \rightarrow \pm\infty} \frac{3x^2 - 1}{x + x^2} = 3$$

$$q = \lim_{x \rightarrow \pm\infty} \frac{\frac{3x^2 - 1}{x}}{x + 1} - 3x = \lim_{x \rightarrow \pm\infty} \frac{3x^2 - 1 - 3x^2 - 3x}{x + 1} = \lim_{x \rightarrow \pm\infty} \frac{-3x - 1}{x + 1} = -3$$

as. se směrnicí $+\infty$ $y = 3x - 3$

1) Určete definiční obor funkce:

$$f(x) = \sqrt{\frac{2x+3}{x}} + \ln(-x^2 + 2x + 8)$$

2) Vypočítejte limitu

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

3) Určete intervaly, na kterých je funkce $f(x) = x^2 + \frac{1}{x}$ konvexní či konkávní, najděte inflexní body.

4) Najděte rovnici tečny a normály ke grafu funkce $f(x) = e^{-x^2+1}$ v bodě T[1;?].

① $\frac{2x+3}{x} \geq 0 \quad x \neq 0$

$\begin{array}{c} \oplus \\ \hline + \end{array}$	$\begin{array}{c} - \\ \hline - \end{array}$	$\begin{array}{c} \oplus \\ \hline + \end{array}$
$-\frac{3}{2}$	0	-2 4

$-x^2 + 2x + 8 > 0$

$-(x^2 - 2x - 8) > 0$

$-(x-4)(x+2) > 0$

$\begin{array}{c} \oplus \\ \hline - \end{array}$

$Df = (-2; -\frac{3}{2}) \cup (0, 4)$

② $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}}{1} = \lim_{x \rightarrow 0} \frac{1}{2\sqrt{1+x}} = \frac{1}{2}$

③ $f' = 2x + (-x^{-2})$

$f'' = 2 + 2x^{-3} = 2 + \frac{2}{x^3}$

$Df = R \setminus \{0\}$

$2 + \frac{2}{x^3} = 0$

$2x^3 + 2 = 0 \quad \text{konvexn' } (-\infty, -1], (0, \infty)$

$2x^3 = -2$

$x^3 = -1 \quad \text{konkavn' } [-1, 0]$

$x = -1 \quad \text{infl. bod } [-1, 0]$

④ $x_0 = 1 \quad y_0 = e^{-1+1} = 1$

$f' = e^{-x^2+1} \cdot (-2x)$

$f'(1) = e^{-1+1} \cdot (-2) = -2$

1: $y = -2(x-1)+1$

$y = -2x + 3$

3: $y = -\frac{1}{2}(x-1)+1$

$y = \frac{1}{2}x + \frac{1}{2}$

1) Urcete definicní obor funkce:

$$f(x) = \frac{\ln(x^2 + 2x)}{\sqrt{x^2 - 5x - 6}}$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow 0} \frac{e^{2x} - x - 1}{3x}$$

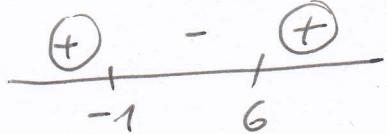
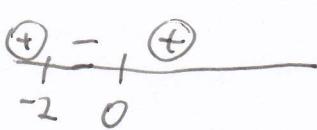
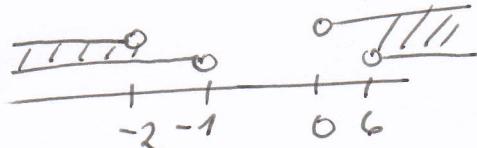
3) Urcete intervaly, na kterých je funkce $f(x) = 3x^5 - 5x^4 + 4$ konvexní ci konkávní, najdete inflexní body.4) Určete rovnice všech asymptot funkce $f(x) = xe^{-x}$.

$$\textcircled{1} \quad x^2 + 2x > 0$$

$$x(x+2) > 0$$

$$x^2 - 5x - 6 > 0$$

$$(x+1)(x-6) > 0$$



$$Df = (-\infty, -2) \cup (6, \infty)$$

$$\textcircled{2} \quad \lim_{x \rightarrow 0} \frac{e^{2x} - x - 1}{3x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^{2x} \cdot 2 - 1}{3} = \frac{2-1}{3} = \underline{\underline{\frac{1}{3}}}$$

$$\textcircled{3} \quad f' = 15x^4 - 20x^3$$

$$Df = \mathbb{R}$$



$$f'' = 60x^3 - 60x^2$$

$$60x^3 - 60x^2 = 0$$

$$60x^2(x-1) = 0$$

$$x=0 \quad x=1$$

konkávní $(-\infty; 0), (0, 1)$
konvexní $(1, \infty)$
infl. bod $[1; 2]$

$$\textcircled{4} \quad Df = \mathbb{R} \rightarrow \text{bez směrnice nejsou}$$

$$k_1 = \lim_{x \rightarrow +\infty} \frac{xe^{-x}}{x} = \lim_{x \rightarrow +\infty} e^{-x} = 0$$

$$k_2 = \lim_{x \rightarrow -\infty} \frac{xe^{-x}}{x} = \lim_{x \rightarrow -\infty} e^{-x} = +\infty \text{ r } -\infty \text{ as. neexistuje}$$

$$g = \lim_{x \rightarrow +\infty} xe^{-x} - 0 = \lim_{x \rightarrow \infty} \frac{x}{e^x} = \left[\frac{c}{c} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

$$\text{as. r } +\infty : g = 0$$

1) Urcete definicní obor funkce:

$$f(x) = \frac{\ln(x^2 - x - 6)}{\sqrt{4x - x^2}}$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow \infty} \frac{\ln x - 2}{x + 1}$$

3) Urcete intervaly, na kterých je funkce $f(x) = -2 + 12x - x^3$ rostoucí ci klesající a najdete lokální extrémy.

4) Urcete rovnici tecny ke grafu funkce $f(x) = x \ln x$ v bode T[1;?] a pomocí diferenciálu vypocítejte približnou hodnotu $f(1,01)$.

① $x^2 - x - 6 > 0$

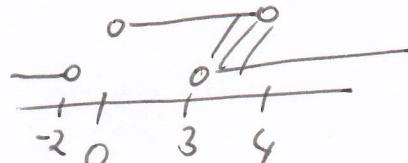
$$(x-3)(x+2) > 0$$

$$\begin{array}{c} \oplus \\ \hline - \end{array} \quad \begin{array}{c} - \\ \hline 1 \end{array} \quad \begin{array}{c} \oplus \\ \hline + \end{array}$$

$$4x - x^2 > 0$$

$$x(4-x) > 0$$

$$\begin{array}{c} - \\ \hline 0 \end{array} \quad \begin{array}{c} \oplus \\ \hline + \end{array} \quad \begin{array}{c} - \\ \hline 4 \end{array}$$



$$Df = (3, 4)$$

② $\lim_{x \rightarrow \infty} \frac{\ln x - 2}{x + 1} = \left[\frac{\infty}{\infty} \right] \stackrel{\text{l'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$

③ $f' = 12 - 3x^2$

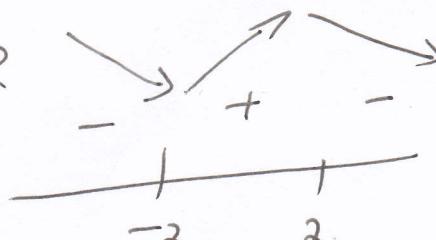
$$12 - 3x^2 = 0$$

$$12 = 3x^2$$

$$4 = x^2$$

$$\pm 2 = x$$

$$Df = R$$



rost. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

kles. $(-\infty, -2), (-2, 2)$

lok. max. $[2, 14]$

lok. min. $[-2, -18]$

④ $x_0 = 1 \quad y_0 = 1 \cdot \ln 1 = 0$

$$f' = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

$$f'(1) = \ln 1 + 1 = 1$$

$$t: y = 1 \cdot (x-1) + 0$$

$$\underline{y = x-1}$$

$$f(1,01) = 1,01 - 1 = \underline{0,01}$$

1) Urcete definicní obor funkce:

$$f(x) = \ln(x^2 - 4x - 5) + \sqrt{\frac{2x-4}{3x-5}}$$

2) Vypocítejte limitu

$$\lim_{x \rightarrow \infty} \frac{x \ln x}{2x+1}$$

3) Urcete intervaly, na kterých je funkce $f(x) = x^4 + 2x^3 - 12x^2$ konvexní ci konkávní, najdete inflexní body.

4) Najděte všechny asymptoty funkce $f(x) = \frac{x^2 + 2}{1-x}$.

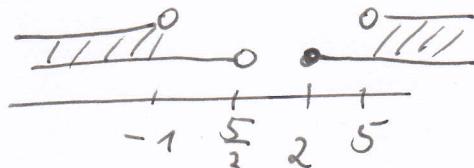
$$\textcircled{1} \quad x^2 - 4x - 5 > 0$$

$$(x-5)(x+1) > 0$$

$$\begin{array}{c|cc|c} + & - & + \\ \hline -1 & & 5 & \end{array}$$

$$\frac{2x-4}{3x-5} \geq 0$$

$$\begin{array}{c|cc|c} + & - & + \\ \hline \frac{5}{3} & & 2 & \end{array}$$

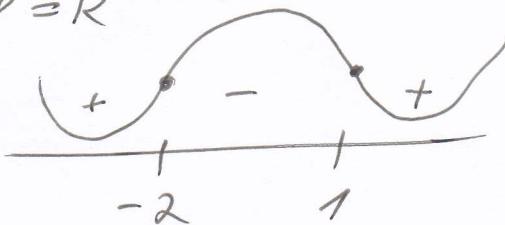


$$Df = (-\infty; -1) \cup (5; +\infty)$$

$$\textcircled{2} \quad \lim_{x \rightarrow \infty} \frac{x \ln x}{2x+1} = \left[\frac{\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\ln x + x \frac{1}{x}}{2} = \lim_{x \rightarrow \infty} \frac{\ln x + 1}{2} = \underline{\underline{\infty}}$$

$$\begin{aligned} \textcircled{3} \quad f' &= 4x^3 + 6x^2 - 24x \\ f'' &= 12x^2 + 12x - 24 \\ 12x^2 + 12x - 24 &= 0 \\ 12(x^2 + x - 2) &= 0 \\ 12(x+2)(x-1) &= 0 \\ x = -2 & \quad x = 1 \end{aligned}$$

$$Df = \mathbb{R}$$



konkávní $(-2, 1)$

konvexní $(-\infty, -2)$, $(1, \infty)$

infl. body $[-2, -48]$
[1, -9]

$$\textcircled{4} \quad Df = \mathbb{R} \setminus \{1\} \quad \lim_{x \rightarrow 1^+} \frac{x^2+2}{1-x} = \left[\frac{-3}{0-} \right] = -\infty \Rightarrow \text{as. bes. směrnice } x=1$$

$$k = \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2+2}{1-x}}{x} = \lim_{x \rightarrow \pm\infty} \frac{x^2+2}{x-x^2} = -1$$

$$g = \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2+2}{1-x}}{x} + x = \lim_{x \rightarrow \pm\infty} \frac{x^2+2+x-x^2}{1-x} = \lim_{x \rightarrow \pm\infty} \frac{2+x}{1-x} = -1$$

as. se směrnicí $x \neq \pm\infty$

$$y = -x - 1$$

1) Urcete definicní obor funkce:

$$f(x) = \ln \frac{3-x}{-3-x} + \sqrt{-x^2 - 3x + 4}$$

2) Vypocítejte limitu

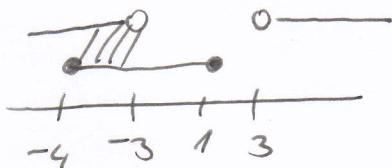
$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x}$$

3) Urcete intervaly, na kterých je funkce $f(x) = \frac{x}{1+x^2}$ rostoucí ci klesající a najdete lokální extrémy.

4) Napište rovnici tečny a normály ke grafu funkce $f(x) = x^3 + 2x$ v bode T[1;?]

$$\textcircled{1} \quad \frac{3-x}{-3-x} > 0$$

$$-x^2 - 3x + 4 \geq 0$$



$$\begin{array}{c} \textcircled{+} \\ + \end{array} \quad - \quad \begin{array}{c} \textcircled{+} \\ + \end{array}$$

$$\begin{array}{c} -3 \\ 3 \end{array}$$

$$-(x^2 + 3x - 4) \geq 0$$

$$-(x-1)(x+4) \geq 0$$

$$\begin{array}{c} - \\ + \end{array} \quad \begin{array}{c} \textcircled{+} \\ + \end{array} \quad \begin{array}{c} - \\ - \end{array}$$

$$\begin{array}{c} -4 \\ 1 \end{array}$$

$$Df = (-4, -3)$$

$$\textcircled{2} \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x} = \left[\begin{matrix} 0 \\ 0 \end{matrix} \right] \stackrel{\text{L'H}}{\Rightarrow} \lim_{x \rightarrow 0} \frac{e^x}{\cos 2x \cdot 2} = \frac{1}{1 \cdot 2} = \underline{\underline{\frac{1}{2}}}$$

$$\textcircled{3} \quad f' = \frac{1 \cdot (1+x^2) - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$1-x^2 = 0 \quad Df = R$$

$$1 = x^2$$

$$\pm 1 = x$$

$$\begin{array}{c} + \\ - \end{array} \quad \begin{array}{c} + \\ - \end{array}$$

$$\begin{array}{c} -1 \\ 1 \end{array}$$

kles. $(-\infty, -1], [1, \infty)$
 rost. $(-1, 1)$
 lok. min. $\left[-1, -\frac{1}{2}\right]$
 lok. max. $\left[1, \frac{1}{2}\right]$

$$\textcircled{4} \quad x_0 = 1 \quad y_0 = 1 + 2 \cdot 1 = 3$$

$$l: y = 5(x-1) + 3 \quad m: y = -\frac{1}{5}(x-1) + 3$$

$$f' = 3x^2 + 2 \quad \underline{\underline{y = 5x - 2}}$$

$$f'(1) = 5 \quad \underline{\underline{y = -\frac{1}{5}x + \frac{16}{5}}}$$