2. zápočtová písemka – skupina A

1) Určete inverzní funkci k funkci $f(x) = \log_7(x-3)+2$. A dále určete definiční obory a obory hodnot obou funkcí.

2) Vypočítejte derivaci funkce $f(x) = \dfrac{\log 3x}{\cos x^2}$.

3) Určete intervaly,na nichž je funkce $f(x) = x \cdot e^{-x}$ konvexní, konkávní a určete inflexní bod.

4) Vypočítejte determinant:

$$\begin{vmatrix} 1 & -1 & 0 & 2 \\ 1 & 0 & 3 & -1 \\ 2 & -1 & 0 & 0 \\ 0 & 1 & 3 & -1 \end{vmatrix}$$

2. zápočtová písemka – skupina B

1) Určete definiční obor funkce $f(x) = \sqrt{\dfrac{x+2}{1-x}} - \arcsin\dfrac{2x-1}{3}$.

2) Určete inverzní funkci k funkci $f: y = \cot g(x - \dfrac{\pi}{2}) + 2$. A dále určete definiční obory a obory hodnot obou funkcí.

3) Určete intervaly, na nichž je funkce $f(x) = x^2 \cdot e^{\frac{1}{x}}$ rostoucí, klesající a určete lokální extrémy.

4) Vypočítejte $A \cdot B \cdot C$, kde

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

2. zápočtová písemka – skupina C

1) Určete definiční obor funkce $f(x) = \dfrac{3x-5}{\log(3x-4)} + \arccos\left(\dfrac{x}{4} - 1\right)$.

2) Vypočítejte derivaci funkce $f(x) = \sqrt{\cos x \cdot e^{-x}}$.

3) Napište rovnici tečny a normály funkce $f(x) = \dfrac{x^2+1}{x-3}$ v bodě T [4;?].

4) Vypočítejte $\left(2A + B^T\right) \cdot B$, kde

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 3 \end{pmatrix}$$

2. zápočtová písemka – skupina D

1) Určete definiční obor funkce $f(x) = \dfrac{\sin x - 1}{\sqrt{x^2 + 3x - 10}} + \log \dfrac{3-x}{x+3}$.

2) Určete intervaly, na nichž je funkce $f(x) = \dfrac{x^2+1}{x^2-1}$ rostoucí, klesající a určete lokální extrémy.

3) Vypočítejte derivaci funkce $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$ a výsledek zjednodušte.

4) Vypočítejte:

$$\begin{vmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & -1 & -1 \end{vmatrix}$$

A

1)

$$y = \log_7(x-3) + 2 \quad \rightarrow \quad x-3>0$$
$$x>3$$

$$f^{-1}: \quad x = \log_7(y-3)+2 \qquad Df = (3;\infty) = Hf^{-1}$$

$$x-2 = \log_7(y-3)$$

$$7^{x-2} = y-3$$

$$f^{-1}: \quad \boxed{7^{x-2}+3 = y} \qquad Df^{-1} = R = Hf$$

2)

$$f(x) = \frac{\log 3x}{\cos x^2}$$

$$f'(x) = \frac{\frac{1}{3x\cdot \ln 10}\cdot 3\cdot \cos x^2 - \log 3x \cdot (-\sin x^2)\cdot 2x}{\cos^2 x^2} =$$

$$= \frac{\frac{\cos x^2}{x \ln 10} + 2x\log 3x\cdot \sin x^2}{\cos^2 x^2}$$

3)

$$f(x) = x\cdot e^{-x}$$

$$Df = R$$

$$f' = 1\cdot e^{-x} + x\cdot e^{-x}\cdot(-1) = e^{-x} - xe^{-x} = e^{-x}(1-x)$$

$$f'' = -e^{-x}\cdot(1-x) + e^{-x}\cdot(-1) = e^{-x}(-1+x-1)$$
$$= e^{-x}(x-2)$$

$$e^{-x}\cdot(x-2) = 0$$

znaménka f''

$-$ $+$ (2)

$$e^{-x} = 0 \qquad x-2=0$$
$$x = 2$$

nelze

! $\boxed{e^{\text{cokoliv}} > 0}$

konkávní $(-\infty; 2\rangle$

konvexní $\langle 2; \infty)$

inflexní bod $x = 2$

4)

$$\begin{vmatrix} 1 & -1 & 0 & 2 \\ 1 & 0 & 3 & -1 \\ 2 & -1 & 0 & 0 \\ 0 & 1 & 3 & -1 \end{vmatrix} =$$

$$= 2\cdot(-1)^{3+1}\cdot \begin{array}{c} \begin{vmatrix} -1 & 0 & 2 \\ 0 & 3 & -1 \\ 1 & 3 & -1 \end{vmatrix} \\ \begin{matrix} -1 & 0 & 2 \\ 0 & 3 & -1 \end{matrix} \end{array} - 1\cdot(-1)^{3+2}\cdot \begin{array}{c} \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & -1 \\ 0 & 3 & -1 \end{vmatrix} \\ \begin{matrix} 1 & 0 & 2 \\ 1 & 3 & -1 \end{matrix} \end{array} + 0 + 0 =$$

$$= 2\cdot\left[3+0+0-(6+3+0)\right] + \left[-3+6+0-(0-3+0)\right] =$$

$$= 2\cdot(-6) + 6 = \underline{\underline{-6}}$$

B

1) $f(x) = \sqrt{\frac{x+2}{1-x}} - \arcsin \frac{2x-1}{3}$

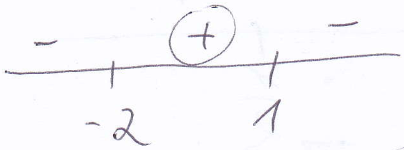
$\downarrow$

$\frac{x+2}{1-x} \geq 0 \qquad 1-x \neq 0$

$\boxed{1 \neq x}$

$x+2=0 \qquad 1-x=0$

$x=-2 \qquad x=1$



$\langle -2; 1)$

$\downarrow$

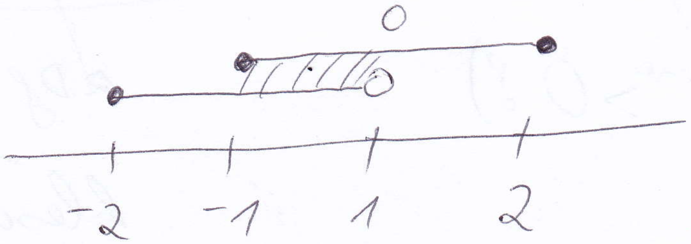
$-1 \leq \frac{2x-1}{3} \leq 1 \quad /\cdot 3$

$-3 \leq 2x-1 \leq 3 \quad /+1$

$-2 \leq 2x \leq 4 \quad /:2$

$-1 \leq x \leq 2$

$\boxed{\langle -1; 2\rangle}$



$\underline{\underline{Df = \langle -1; 1)}}$

2) $y = \text{cotg}\left(x - \frac{\pi}{2}\right) + 2$

$f^{-1}: \quad x = \text{cotg}\left(y - \frac{\pi}{2}\right) + 2$

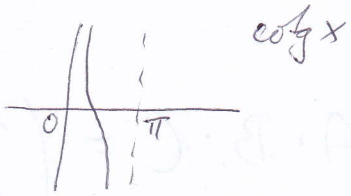
$x - 2 = \text{cotg}\left(y - \frac{\pi}{2}\right)$

$\text{arccotg}(x-2) = y - \frac{\pi}{2}$

$f^{-1}: \boxed{\text{arccotg}(x-2) + \frac{\pi}{2} = y}$

$Df^{-1} = R = Hf$

$\rightarrow$



$0 < x - \frac{\pi}{2} < \pi \quad /+\frac{\pi}{2}$

$\frac{\pi}{2} < x < \frac{3}{2}\pi$

$Df = \left(\frac{\pi}{2}; \frac{3}{2}\pi\right) = Hf^{-1}$

3) $f(x) = x^2 \cdot e^{\frac{1}{x}}$

$\frac{1}{x} = x^{-1}$

sedy derivace je

$Df = R - \{0\}$

$f' = 2x \cdot e^{\frac{1}{x}} + x^2 \cdot e^{\frac{1}{x}} \cdot (-x^{-2})$

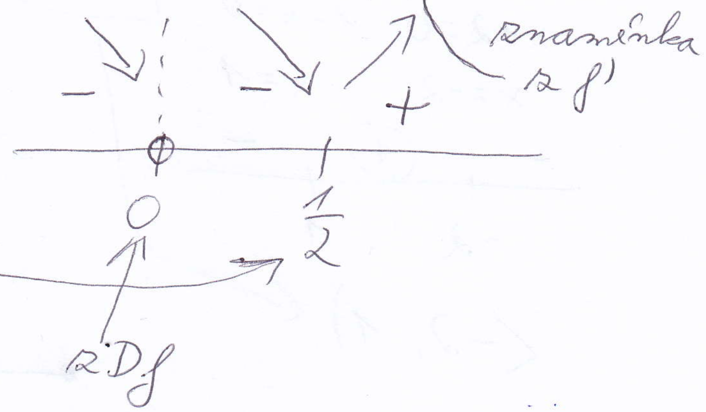
$= e^{\frac{1}{x}}\left(2x + x^2 \cdot \left(-\frac{1}{x^2}\right)\right) = e^{\frac{1}{x}}(2x - 1)$

$e^{\frac{1}{x}}(2x-1) = 0$

$e^{\frac{1}{x}} \neq 0$ není ($e^{\text{cokoliv}} > 0$!)

$2x - 1 = 0$

$x = \frac{1}{2}$



klesající $(-\infty, 0), (0, \frac{1}{2}\rangle$

rostoucí $\langle \frac{1}{2}, \infty)$

lokální min. $x = \frac{1}{2}$

4)

$$A \cdot B \cdot C = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & -1-3 & 0+1 \\ 2+2 & -2+3 & 0-1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -4 & 1 \\ 4 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} -1\cdot1-4\cdot2+1\cdot0 & -1\cdot(-1)-4\cdot(-1)+1\cdot1 & -1\cdot1-4\cdot0+1\cdot3 \\ 4\cdot1+1\cdot2-1\cdot0 & 4\cdot(-1)+1\cdot(-1)-1\cdot1 & 4\cdot1+1\cdot0-1\cdot3 \end{pmatrix} = \begin{pmatrix} -9 & 6 & 2 \\ 6 & -6 & 1 \end{pmatrix}$$

C

1)
$$f(x)=\frac{3x-5}{\log(3x-4)}+\arccos\left(\frac{x}{4}-1\right)$$

$3x-4>0$

$3x>4$

$x>\frac{4}{3}$

$\left(\frac{4}{3};\infty\right)$

$\log_{10}(3x-4)\neq 0$

$3x-4\neq 10^0$

$3x-4\neq 1$

$3x\neq 5$

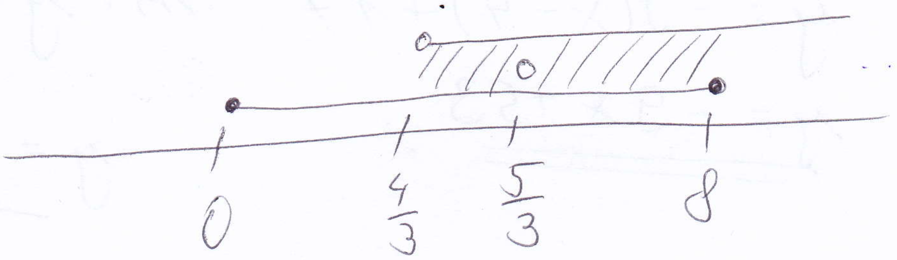
$x\neq\frac{5}{3}$

$-1\leq\frac{x}{4}-1\leq 1$

$-4\leq x-4\leq 4$

$0\leq x\leq 8$

$\langle 0;8\rangle$



$$Df=\left(\frac{4}{3};\frac{5}{3}\right)\cup\left(\frac{5}{3};8\right\rangle$$

$$\text{nebo}=\left(\frac{4}{3};8\right\rangle-\left\{\frac{5}{3}\right\}$$

2)
$$f(x)=\sqrt{\cos x\cdot e^{-x}}$$

$$f'(x)=\frac{1}{2\sqrt{\cos x\cdot e^{-x}}}\cdot\left(-\sin x\cdot e^{-x}+\cos x\cdot e^{-x}\cdot(-1)\right)=$$

$$=\frac{-\sin x\cdot e^{-x}-\cos x\cdot e^{-x}}{2\sqrt{\cos x\cdot e^{-x}}}$$

3) $f(x) = \dfrac{x^2+1}{x-3}$ $\quad T[4; ?]$

$\downarrow$

$x_0 = 4$

$y_0 = \dfrac{4^2+1}{4-3} = \dfrac{17}{1} = 17$

$$f'(x) = \frac{2x(x-3)-(x^2+1)\cdot 1}{(x-3)^2} = \frac{2x^2-6x-x^2-1}{(\cancel{x}-3)^2} = \frac{x^2-6x-1}{(x-3)^2}$$

$$f'(x_0) = f'(4) = \frac{4^2-6\cdot 4-1}{(4-3)^2} = \frac{16-24-1}{1} = -9$$

$t: \; y = -9(x-4)+17$

$\underline{\underline{y = -9x+53}}$

$n: \; y = -\dfrac{1}{-9}(x-4)+17$

$\underline{\underline{y = \dfrac{1}{9}(x-4)+17}}$

4) $A = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 0 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 3 \end{pmatrix}$

$$(2A + B^T)\cdot B = \left[2\cdot\begin{pmatrix} 2 & 1 & -1 \\ 3 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 3 \end{pmatrix}\right]\cdot\begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 3 \end{pmatrix} =$$

$$= \left[\begin{pmatrix} 4 & 2 & -2 \\ 6 & 0 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 3 \end{pmatrix}\right]\cdot\begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & 3 & -2 \\ 5 & 2 & 7 \end{pmatrix}\cdot\begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 5\cdot 1+3\cdot 1-2\cdot 0 & 5(-1)+3\cdot 2-2\cdot 3 \\ 5\cdot 1+2\cdot 1+7\cdot 0 & 5(-1)+2\cdot 2+7\cdot 3 \end{pmatrix}$$

$$\underline{\underline{= \begin{pmatrix} 8 & -5 \\ 7 & 20 \end{pmatrix}}}$$

D

1) $$f(x) = \frac{\sin x - 1}{\sqrt{x^2+3x-10}} + \log\frac{3-x}{x+3}$$

$$x^2+3x-10>0$$

$$(x+5)(x-2)=0$$

$x=-5 \quad x=2$

$+ \quad - \quad +$

$-5 \quad 2$

$$(-\infty,-5)\cup(2,\infty)$$

$$\frac{3-x}{x+3}>0 \qquad x+3\neq 0$$

$$x\neq -3$$

$3-x=0 \quad x+3=0$

$x=3 \quad x=-3$

$- \quad + \quad -$

$-3 \quad 3$

$$(-3;3)$$

$-5 \quad -3 \quad 2 \quad 3$

$$\underline{\underline{Df=(2;3)}}$$

2) $$f(x)=\frac{x^2+1}{x^2-1}$$

$$Df = R-\{-1;1\}$$

$$f'(x)=\frac{2x(x^2-1)-(x^2+1)\cdot 2x}{(x^2-1)^2}=\frac{2x(x^2-1-x^2-1)}{(x^2-1)^2}=\boxed{\frac{-4x}{(x^2-1)^2}}$$

$$\frac{-4x}{(x^2-1)^2}=0 \qquad /\cdot(x^2-1)^2$$

$$-4x=0$$

$$x=0$$

určím znaménka derivace

$+ \quad + \quad - \quad -$

$-1 \quad 0 \quad 1$

$\notin Df \qquad \notin Df$

rostoucí $(-\infty,-1)$, $(-1,0\rangle$

klesající $\langle 0,1)$, $(1,\infty)$

lokální max. $x=0$

3) $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$

$$f'(x) = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x\right) =$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{x}{\sqrt{x^2+1}}\right) =$$

$$= \frac{1}{\cancel{x+\sqrt{x^2+1}}} \cdot \frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}} = \underline{\underline{\frac{1}{\sqrt{x^2+1}}}}$$

4)

$$\begin{vmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & -1 & -1 \end{vmatrix} = 1 \cdot (-1)^{\overset{\text{řádek}}{1}+\overset{\text{sloupec}}{2}} \cdot \begin{vmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & -1 \end{vmatrix} + 0 + 2 \cdot (-1)^{3+2} \cdot \begin{vmatrix} 2 & 0 & -1 \\ 1 & 3 & 1 \\ 2 & -1 & -1 \end{vmatrix}$$

$$+0 = (-1) \cdot \begin{vmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & -1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 0 & -1 \\ 1 & 3 & 1 \\ 2 & -1 & -1 \end{vmatrix} =$$

$$\begin{matrix} 1 & 3 & 1 \\ 0 & 1 & 0 \end{matrix} \qquad \begin{matrix} 2 & 0 & -1 \\ 1 & 3 & 1 \end{matrix}$$

$$= (-1) \cdot [-1 + 0 + 0 - (2 + 0 + 0)] - 2 \cdot [-6 + 1 + 0 - (-6 - 2 + 0)]$$

$$= -1 \cdot (-3) - 2 \cdot 3 = 3 - 6 = \underline{\underline{-3}}$$