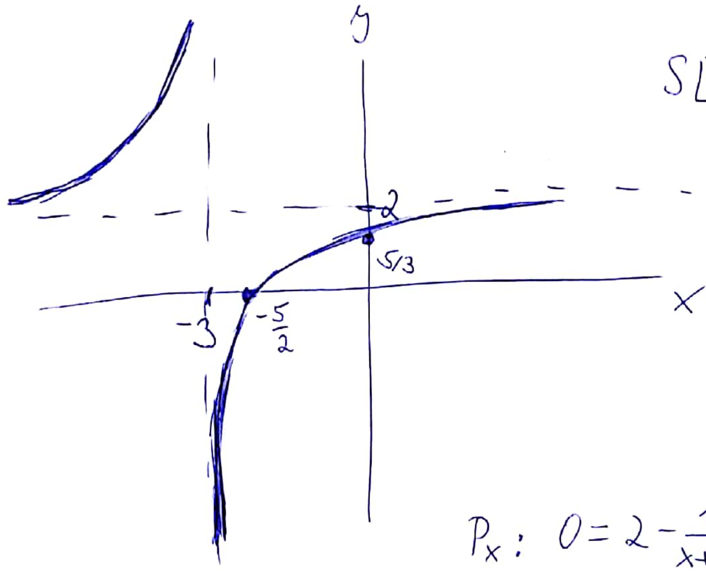
(A)

1) $D_f = R - \{-3\}$

$H_f = R - \{2\}$

je prostá



$P_x: \; 0 = 2 - \frac{1}{x+3}$

$0 = 2x + 6 - 1$

$x = -\frac{5}{2}$

$P_y: \; y = 2 - \frac{1}{0+3}$

$y = \frac{5}{3}$

2) $g(x) = e^{\sqrt{x} \cdot \arcsin x}$

$g'(x) = e^{\sqrt{x} \cdot \arcsin x} \cdot \left(\frac{1}{2\sqrt{x}} \arcsin x + \frac{\sqrt{x}}{\sqrt{1-x^2}} \right)$

3) $D_h = R$

$h' = \frac{3}{4} x^4 - 2x^3 + 3$

$h'' = 3x^3 - 6x^2$

$3x^3 - 6x^2 = 0$

$3x^2(x-2) = 0$

$x = 0 \quad x = 2$

$-$ $-$ $+$

0 2

infl. bod $\underline{\underline{x = 2}}$

(definice inf. b. viz skripta)

4) $|A| = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 13$ regulární

$|B| = \begin{vmatrix} 1 & 2 & 3 \\ -2 & -1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = -4 - 6 - 2 - 3 - 1 + 16 = 0$

singulární

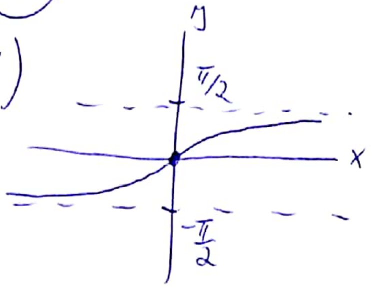
5) $\left(\begin{array}{ccc|c} 1 & 4 & -3 & -2 \\ 1 & -3 & -1 & 0 \\ 2 & 1 & -4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 4 & -3 & -2 \\ 0 & 7 & -2 & -2 \\ 0 & -7 & 2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 4 & -3 & -2 \\ 0 & 7 & -2 & -2 \\ 0 & 0 & 0 & 3 \end{array}\right)$

$\left.\begin{array}{l} h = 2 \\ h_r = 3 \\ n = 3 \end{array}\right\}$ soustava nemá řešení

(Fr. věta viz skripta či tutoriál)

B

1)



$$\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$$

2) $f(x) = \dfrac{5x}{x+3}$

$$f'(x) = \frac{5(x+3) - 5x \cdot 1}{(x+3)^2} = \frac{5x+15-5x}{(x+3)^2} = \frac{15}{(x+3)^2} = 15 \cdot (x+3)^{-2}$$

$$f''(x) = -30(x+3)^{-3} \cdot 1 = \underline{\underline{\frac{-30}{(x+3)^3}}}$$

3) $x_0 = 1$

$y_0 = 5 - \sqrt{2 \cdot 1 - 1} = 5 - 1 = 4$

$n: \; y - y_0 = -\dfrac{1}{f'(x_0)} \cdot (x - x_0)$

$y - 4 = \dfrac{-1}{-1} \cdot (x-1)$

$\underline{\underline{y = x + 3}}$

$f(x) = 5 - \sqrt{2x-1}$

$f'(x) = -\dfrac{1}{2\sqrt{2x-1}} \cdot 2$

$f'(1) = -\dfrac{1}{2 \cdot \sqrt{2 \cdot 1 - 1}} \cdot 2 = -1$

(rozdíl viz skripta či tutoriál)

4) $C + 2X = AX - A$

$2X - AX = -A - C$

$(2J - A) \cdot X = -A - C$

$X = (2J - A)^{-1} \cdot (-A - C)$

$$2J - A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix}$$

$$(2J-A)^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix}$$

$$-A - C = \begin{pmatrix} -3 & -1 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ 1 & -5 \end{pmatrix}$$

$$X = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix} \cdot \begin{pmatrix} -4 & -3 \\ 1 & -5 \end{pmatrix} =$$

$$= \frac{1}{2} \begin{pmatrix} -3 & -8 \\ 11 & 14 \end{pmatrix} = \underline{\begin{pmatrix} -\frac{3}{2} & -4 \\ \frac{11}{2} & 7 \end{pmatrix}}$$

5) $x_1 = \dfrac{|A_1|}{|A|} = \dfrac{18}{-9} = \underline{\underline{-2}}$

$$|A| = \begin{vmatrix} 1 & 1 & -3 \\ 1 & -1 & -2 \\ 2 & 1 & -1 \end{vmatrix} = 1 - 3 - 4 - 6 + 2 + 1 = -9$$

$$|A_1| = \begin{vmatrix} -9 & 1 & -3 \\ -5 & -1 & -2 \\ -7 & 1 & -1 \end{vmatrix} = -9 + 15 + 14 + 21 - 18 - 5 = 18$$

Lze vyřešit pouze, když je matice A regulární, protože $|A| \neq 0$. Jinak by nešlo dosadit do vzorce $x_j = \dfrac{|A_j|}{|A|}$.

c

1) $f(x)=x^2-5x+4$

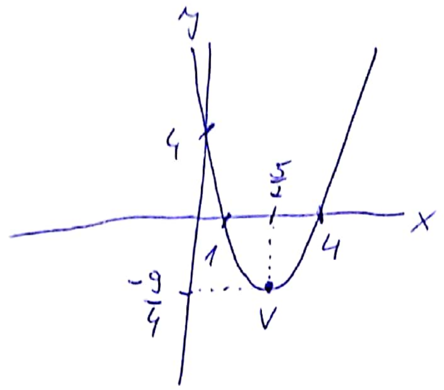
$V\left[\frac{5}{2}; -\frac{9}{4}\right]$

$y_0 = \frac{25}{4} - \frac{25}{2} + 4 = \frac{25-50+16}{4} = -\frac{9}{4}$

Px $\quad 0 = x^2-5x+4$ $\qquad$ Py: $y = 0-5\cdot 0+4=4$

$0=(x-4)(x-1)$

$x=4 \quad x=1$



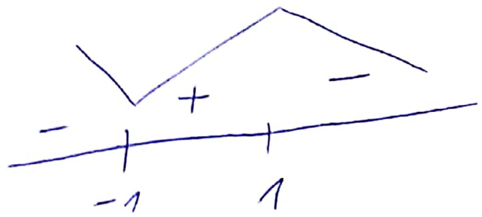
není sudá ani lichá

je omezená zdola

není prostá

2) $f(x)=\frac{x}{1+x^2} \qquad D_f = R$

$f'(x)=\frac{1\cdot(1+x^2)-x\cdot 2x}{(1+x^2)^2} = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$



$\frac{1-x^2}{(1+x^2)^2}=0$

$1-x^2=0$

$\pm 1 = x$

rostoucí $\langle -1;1\rangle$

klesající $(-\infty;1\rangle$, $\langle 1;\infty)$

3) $\lim\limits_{x\to\infty}\frac{\ln(3x)+5x}{e^{2x-1}} = \left[\frac{\infty}{\infty}\right] \overset{LH}{=} \lim\limits_{x\to\infty}\frac{\frac{1}{3x}\cdot 3+5}{e^{2x-1}\cdot 2} = \frac{0\cdot 3+5}{\infty\cdot 2} = \frac{5}{\infty} = \underline{\underline{0}}$

Použili jsme L'Hospitalovo pravidlo.

4) $\begin{pmatrix}-2&1&0&-3\\1&2&1&-2\\-1&0&-1&1\\-3&1&-1&-2\end{pmatrix} \sim \begin{pmatrix}-2&1&0&-3\\0&5&2&-7\\0&2&0&-1\\0&7&2&-8\end{pmatrix} \sim \begin{pmatrix}-2&0&1&-3\\0&2&5&-7\\0&0&2&-1\\0&2&7&-8\end{pmatrix} \sim \begin{pmatrix}-2&0&1&-3\\0&2&5&-7\\0&0&2&-1\\\cancel{0}&\cancel{0}&\cancel{2}&\cancel{-1}\end{pmatrix}$ $\quad h=3$

(Definice viz tutoriál)

5) $\begin{vmatrix}2&1&0&-1\\0&2&1&-2\\-1&0&-1&1\\-3&0&-1&-2\end{vmatrix} = 1\cdot(-1)^{1+2}\cdot\begin{vmatrix}0&1&-2\\-1&-1&1\\-3&-1&-2\end{vmatrix} + 2\cdot(-1)^{2+2}\cdot\begin{vmatrix}2&0&-1\\-1&-1&1\\-3&-1&-2\end{vmatrix} = 1\cdot(-1)\cdot(-1)+2\cdot 1\cdot 8 =$

$= \underline{\underline{17}}$

$0-2-3+6-0-2=-1 \qquad 4-1+0+3+2-0=8$

Čtvercová matice jejíž det se rovná 0 se nazývá singulární.

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