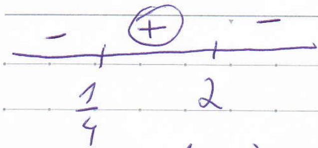
(A)

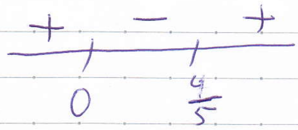
1) $\frac{4x-1}{2-x} > 0$



$D_f = \left(\frac{1}{4}; 2\right)$

2) $5x^2 - 4x \geq 0$

$x(5x-4) \geq 0$



$x \in (-\infty; 0\rangle \cup \langle \frac{4}{5}; \infty)$

3) $\left(\frac{x^2}{y^2} - \frac{x}{y^2}\right) : \frac{(x-1)^2}{y^2} = \frac{x^2-x}{y^2} \cdot \frac{y^2}{(x-1)^2} = \frac{x(x-1)}{(x-1)^2} = \frac{x}{x-1}$

$x \neq 1 \quad y \neq 0$

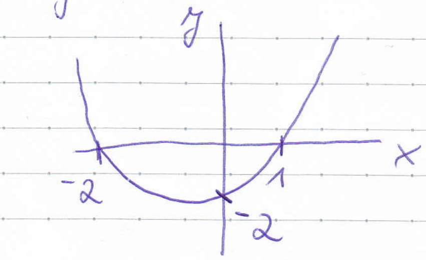
4) $\sqrt{x \cdot x^{\frac{1}{3}}} : x^{\frac{2}{3}} = \left(x^{\frac{4}{3}}\right)^{\frac{1}{2}} : x^{\frac{2}{3}} = x^{\frac{2}{3}} : x^{\frac{2}{3}} = 1 \qquad x > 0$

5)

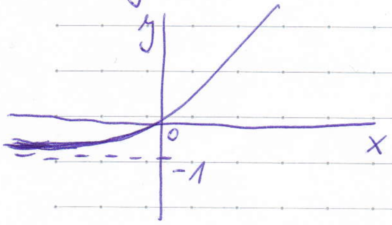
není sudá

neplatí $f(x) = f(-x)$

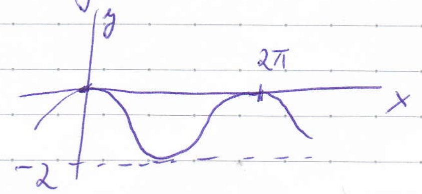
6) $y = x^2 + x - 2 = (x+2)(x-1)$



7) $y = e^x - 1$



8) $y = \cos x - 1$



9) $\cos\left(x + \frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2} \qquad y = x + \frac{\pi}{2}$

$\cos y = -\frac{\sqrt{3}}{2}$

$y_1 = \frac{5}{6}\pi + 2k\pi \qquad y_2 = \frac{7}{6}\pi + 2k\pi$

$x + \frac{\pi}{2} = \frac{5}{6}\pi + 2k\pi \qquad x + \frac{\pi}{2} = \frac{7}{6}\pi + 2k\pi$

$x_1 = \frac{1}{3}\pi + 2k\pi \qquad x_2 = \frac{2}{3}\pi + 2k\pi$

10) $5^{-x+1} - 5^{x-1} = 120$

$5^x(5 - 5^{-1}) = 120$

$5^x\left(5 - \frac{1}{5}\right) = 120$

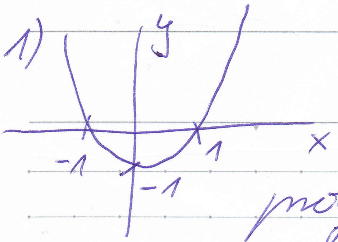
$5^x \cdot \frac{24}{5} = 120$

$5^x = \frac{5 \cdot 120}{24}$

$5^x = 25$

$\boxed{x = 2}$

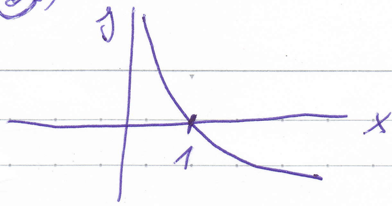
(B)

1) 

není prostá!

protože např.
$f(1)=f(-1)$

3) $y=-\log_5 -x$

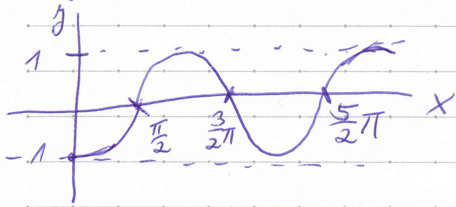


2) $\dfrac{x^3+x^2y+xy^2+y^3}{y^2} : \dfrac{x^4-y^4}{x^2y^2} = \dfrac{x^2(x+y)+y^2(x+y)}{y^2}$

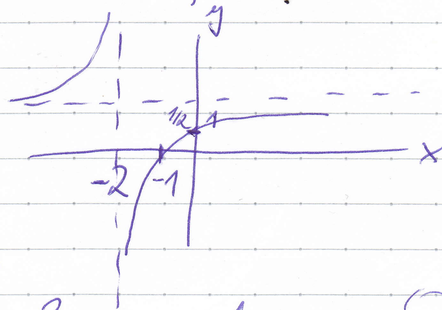
$\cdot \dfrac{x^2y^2}{x^4-y^4} = \dfrac{\cancel{(x^2+y^2)}(x+y)\cdot x^2}{(x^2-y^2)\cancel{(x^2+y^2)}} = \dfrac{x^2}{x-y}$

$x\neq 0$
$y\neq 0$
$x\neq \pm y$

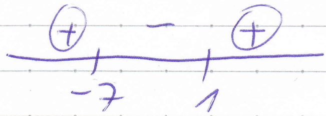
4) $y=\sin\left(x-\frac{\pi}{2}\right)$



5) $S[-2;1]$

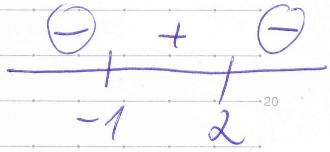


6) $x^2+6x-7\geq 0$
$(x+7)(x-1)\geq 0$



$Df=(-\infty,-7\rangle \cup \langle 1,\infty)$

7) $\dfrac{3}{x+1}<1$



$\dfrac{3}{x+1}-1<0$

$\dfrac{3-x-1}{x+1}<0$

$\dfrac{2-x}{x+1}<0$

$x\in(-\infty,-1)\cup(2;\infty)$

8) $\left(\dfrac{x\cdot x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{3}{2}} = \left(\dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{3}{2}} =$

$= \left(x^{\frac{5}{6}}\right)^{\frac{3}{2}} = x^{\frac{5}{4}} = \sqrt[4]{x^5}$

$x>0$

9) $\sin(2x-\pi)=-1$ $\quad y=2x-\pi$
$\sin y=-1$
$y=\frac{3}{2}\pi+2k\pi$
$2x-\pi=\frac{3}{2}\pi+2k\pi$
$2x=\frac{5}{2}\pi+2k\pi$
$x=\frac{5}{4}\pi+k\pi$

10) $\log\dfrac{x}{2-x}=1$

$x>0$
$x<2$

$\dfrac{x}{2-x}=10$

$x=20-10x$

$11x=20$

$x=\dfrac{20}{11}$

(c)

1) $\frac{3-2x}{x+1} \geq 0$

$-$ $\quad$ $+$ $\quad$ $-$

-1 $\quad$ $\frac{3}{2}$

$D_f = (-1; \frac{3}{2}\rangle$

2) $-x^2-2x+15 \geq 0$

$x^2+2x-15 \leq 0$

$(x+5)(x-3) \leq 0$

$+$ $\quad$ $-$ $\quad$ $+$

-5 $\quad$ 3

$x \in \{-5; -4; -3; -2; -1; 0; 1; 2; 3\}$

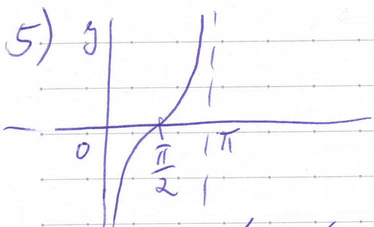
3) $\frac{\cancel{2k^2}-k-2k+\cancel{1}-(\cancel{2k^2}+k+2k+\cancel{1})}{(k+1)(k-1)} : \frac{k-1+1}{k-1} = \frac{-6k}{(k+1)\cancel{(k-1)}} \cdot \frac{\cancel{k-1}}{k} =$

$= \boxed{\frac{-6}{k+1}}$ $\qquad k \neq \pm 1$, $k \neq 0$

4) $\left(\frac{x^2 \cdot x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^{\frac{1}{3}} = \left(\frac{x^{\frac{5}{2}}}{x^{\frac{1}{3}}}\right)^{\frac{1}{3}} = \left(x^{\frac{13}{6}}\right)^{\frac{1}{3}} = x^{\frac{13}{18}} = \sqrt[18]{x^{13}}$

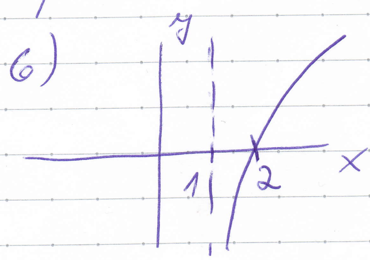
$x > 0$

5)

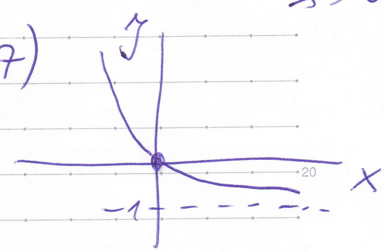


je rostoucí

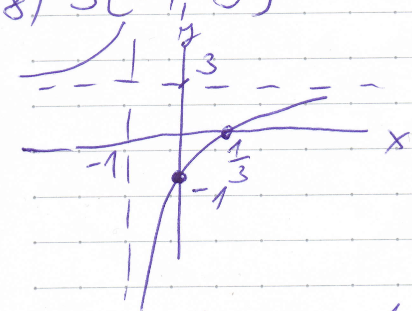
6)



7)



8) $S[-1; 3]$



9) $\cos(2x - \frac{\pi}{2}) = -\frac{1}{2}$

$y = 2x - \frac{\pi}{2}$

$\cos y = -\frac{1}{2}$

$y_1 = \frac{2}{3}\pi + 2k\pi$ $\qquad$ $y_2 = \frac{4}{3}\pi + 2k\pi$

$2x - \frac{\pi}{2} = \frac{2}{3}\pi + 2k\pi$ $\qquad$ $2x - \frac{\pi}{2} = \frac{4}{3}\pi + 2k\pi$

$2x = \frac{7}{6}\pi + 2k\pi$ $\qquad$ $2x = \frac{11}{6}\pi + 2k\pi$

$x_1 = \frac{7}{12}\pi + k\pi$ $\qquad$ $x_2 = \frac{11}{12}\pi + k\pi$

10) $4^{x+1} - 8 \cdot 4^{x-1} = 32$

$4^x(4 - \frac{8}{4}) = 32$

$4^x = 16$

$\boxed{x = 2}$

D

NO.

DATE

1) $1-9x^2>0$

$(1-3x)(1+3x)>0$

$-\quad +\quad -$

$-\frac{1}{3} \qquad \frac{1}{3}$

$Df = \left(-\frac{1}{3}; \frac{1}{3}\right)$

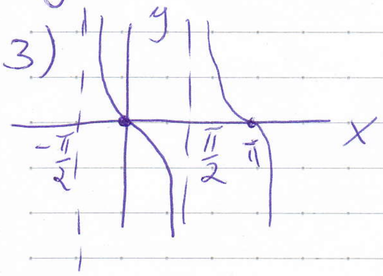
2) $\dfrac{3-2x}{x+1} \leq 0$

$-\quad +\quad -$

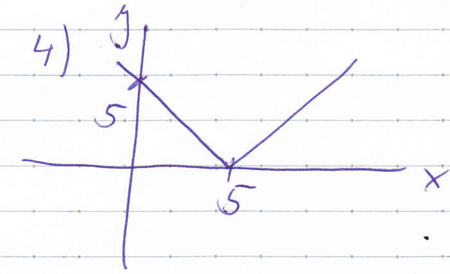
$-1 \qquad \frac{3}{2}$

$x \in (-\infty; -1) \cup \langle \frac{3}{2}; \infty)$

3)

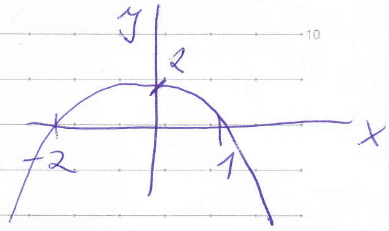


4)



5) $y=-x^2-x+2$

$=-(x+2)(x-1)$



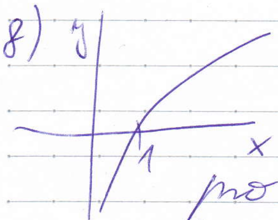
6) $\dfrac{2y^2-y+2y-1+1}{2y-1} \cdot \dfrac{2y^2+y-2y-1+1}{2y+1} =$

$= \dfrac{2y^2+y}{2y-1} \cdot \dfrac{2y^2-y}{2y+1} = \dfrac{y\cancel{(2y+1)}}{\cancel{(2y-1)}} \cdot \dfrac{y\cancel{(2y-1)}}{\cancel{(2y+1)}} = y^2$

$y \neq \pm \frac{1}{2}$

7) $\dfrac{x^2}{x^{\frac{2}{3}}} \cdot \dfrac{x^{\frac{1}{2}}}{x} = \dfrac{x \cdot x^{\frac{1}{2}}}{x^{\frac{2}{3}}} = \dfrac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}} = x^{\frac{5}{6}} = \sqrt[6]{x^5} \qquad x>0$

8)



je prostá

pro žádné dvě x_1, x_2 neplatí:

$f(x_1)=f(x_2)$

9) $tg(\pi-2x)=1$

$y=\pi-2x$

$tg\, y = 1$

$y = \frac{\pi}{4} + k\pi$

$\pi-2x = \frac{\pi}{4}+k\pi$

$-2x = -\frac{3}{4}\pi + k\pi$

$x = \frac{3}{8}\pi - \frac{k\pi}{2}$

10) $\dfrac{3+\log_7 x}{2-\log_7 x} = 4 \qquad x>0, \; x \neq 49$

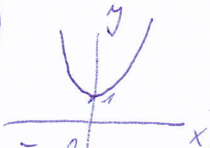
$3+\log_7 x = 8-4\log_7 x$

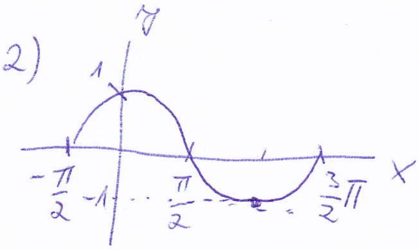
$5\log_7 x = 5$

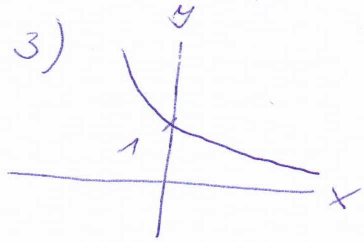
$\log_7 x = 1$

$\boxed{x=7}$

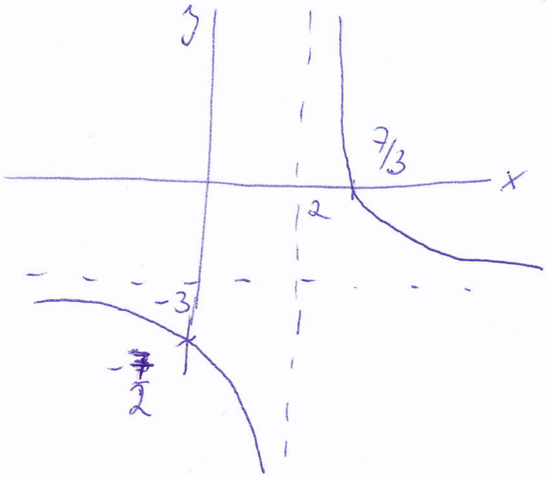
E

1)  je sudá, pro všechna x platí $f(x)=f(-x)$

2) 

3) 

4) $S[2; -3]$



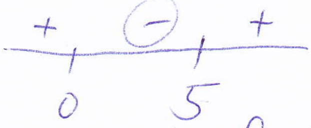
5) $x-3>0 \qquad 9-2x>0$

$x>3 \qquad \frac{9}{2}>x$

$Df = \left(3; \frac{9}{2}\right)$

6) $x^2-5x \leq 0$

$x(x-5) \leq 0$



$x \in \{1; 2; 3; 4; 5\}$

7) $\frac{5x-4x+4}{20} : \frac{5x+5-3x+3}{30} = \frac{x+4}{20} \cdot \frac{30}{2x+8} = \frac{3(x+4)}{2\cdot 2(x+4)} = \frac{3}{4}$

$x \neq -4$

8) $x^{\frac{1}{3}} \cdot \left(\frac{x^{\frac{1}{6}}}{x \cdot x^{\frac{2}{3}}}\right)^2 = \frac{x^{\frac{1}{3}} \cdot x^{\frac{1}{3}}}{x^2 \cdot x^{\frac{4}{3}}} = \frac{1}{x^2 \cdot x^{\frac{2}{3}}} = \frac{1}{x^{\frac{8}{3}}} = \frac{1}{\sqrt[3]{x^8}}$

9) $y = x + \frac{\pi}{2}$

$\sin y = \frac{\sqrt{2}}{2}$

$y_1 = \frac{\pi}{4} + 2k\pi \qquad y_2 = \frac{3}{4}\pi + 2k\pi$

$x + \frac{\pi}{2} = \frac{\pi}{4} + 2k\pi \qquad x + \frac{\pi}{2} = \frac{3}{4}\pi + 2k\pi$

$x = -\frac{\pi}{4} + 2k\pi \qquad x = \frac{\pi}{4} + 2k\pi$

10) $\log_4 x + \frac{4}{\log_4 x} = 5 \quad | \cdot \log_4 x \qquad x>0$

$\log_4^2 x + 4 = 5\log_4 x$

$y = \log_4 x$

$y^2 - 5y + 4 = 0$

$(y-4)(y-1) = 0$

$y = 4 \qquad y = 1$

$\log_4 x = 4 \qquad \log_4 x = 1$

$x = 4^4 \qquad x = 4$

podm.: $x>0$

$\log_4 x \neq 0 \rightarrow x \neq 1$