Určete definiční obory uvedených funkcí:

$$f_1(x) = x^2 + 3x - 1$$

$$f_2(x) = \frac{x+1}{x-3}$$

$$f_3(x) = \frac{x}{13x^2 + 10x - 3}$$

$$f_4(x) = \frac{x^2+1}{x^2+x+1}$$

$$f_5(x) = \frac{1}{|x+3| - 4}$$

$$f_6(x) = \sqrt{x+2}$$

$$f_7(x) = \frac{1}{\sqrt{2x^2+3x-2}}$$

$$f_8(x) = \sqrt{\frac{x+2}{4x-6}}$$

$$f_9(x) = \frac{\sqrt{x+2}}{\sqrt{4x-6}}$$

$$f_{10}(x) = \sqrt{x+4} + \sqrt{\frac{7}{10-x}}$$

$$f_{11}(x) = \log(x-3)$$

$$f_{12}(x) = \frac{1}{\log_2(x+4) - 3}$$

$$f_{13}(x) = \sqrt{\log_{\frac{1}{3}}(2x+1)}$$

$$f_{14}(x) = \sqrt{\log_5 x + 1}$$

Řešení

$D(f_1) = \mathsf{R}$

$D(f_2) = \mathsf{R} - \{3\}$

$D(f_3) = \mathsf{R} - \{-1; \frac{3}{13}\}$

$D(f_4) = \mathsf{R}$

$D(f_5) = \mathsf{R} - \{-7; 1\}$

$D(f_6) = \langle -2; \infty)$

$D(f_7) = (-\infty; -2) \cup (\frac{1}{2}; \infty)$

$D(f_8) = (-\infty; -2\rangle \cup (\frac{3}{2}; \infty)$

$D(f_9) = (\frac{3}{2}; \infty)$

$D(f_{10}) = \langle -4; 10)$

$D(f_{11}) = (3; \infty)$

$D(f_{12}) = (-4; \infty) - \{4\}$

$D(f_{13}) = (-\frac{1}{2}; 0\rangle$

$D(f_{14}) = \langle \frac{1}{5}; \infty)$

Určete definiční obor funkce:

a. $f(x) = \log(3x-1)$ $\qquad D(f) = \left(\frac{1}{3}; \infty\right)$

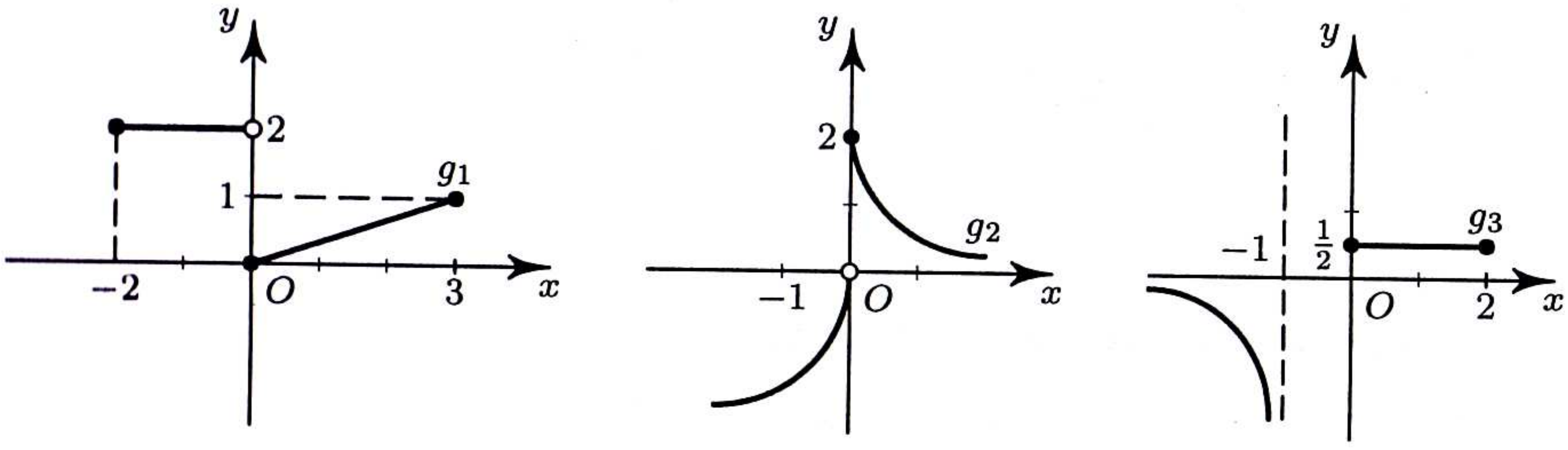
b. $f(x) = \dfrac{\log(3x-2)}{x^2 - x - 2}$ $\qquad D(f) = \left(\frac{2}{3}; \infty\right) - \{2\}$

c. $f(x) = \sqrt{\log(3x^2 - 2x)}$ $\qquad D(f) = \left(-\infty; -\frac{1}{3}\right\rangle \cup \langle 1; +\infty)$

d. $f(x) = \sqrt{x-1} + \log(2-x)$ $\qquad D(f) = \langle 1; 2)$

e. $f(x) = \dfrac{1}{\sqrt{(2x-1)(x+3)}}$ $\qquad D(f) = (-\infty; -3) \cup \left(\frac{1}{2}; +\infty\right)$

Na obr. 2 jsou grafy funkcí g_1, g_2, g_3:



Obr. 2

a) Určete definiční obor a obor funkčních hodnot pro funkce g_1, g_2, g_3.

b) Určete čísla x_1, x_2, x_3 z definičního oboru funkce g_1 tak, aby platilo:

$g_1(x_1) = 0{,}5$ $\qquad g_1(x_2) = 2$ $\qquad g_1(x_3) = -3$

Řešení

a) $\mathrm{D}(g_1) = \langle -2; 3 \rangle$; $\qquad \mathrm{D}(g_2) = \mathrm{R}$; $\qquad \mathrm{D}(g_3) = (-\infty; -1) \cup \langle 0; 2 \rangle$;

$\mathrm{H}(g_1) = \langle 0; 1 \rangle \cup \{2\}$; $\qquad \mathrm{H}(g_2) = (-\infty; 2\rangle - \{0\}$; $\qquad \mathrm{H}(g_3) = (-\infty; 0) \cup \{\frac{1}{2}\}$.

b) $x_1 = 1{,}5$; $\quad x_2 \in \langle -2; 0)$; $\quad x_3 \in \emptyset$.

ÚLOHY K ŘEŠENÍ

Úloha Najděte definiční obory zadaných funkcí

(a) $y = \sqrt{1-x}$ (b) $y = \sqrt{1+x^2}$ (c) $y = \ln(2-x) + \ln(x+1)$

(d) $y = \ln(4-2x)$ (e) $y = \sqrt{x^2+4x+3}$ (f) $y = \dfrac{1}{x} + \dfrac{1}{x+1}$

(g) $y = \dfrac{\sqrt{x}+2}{\sqrt{x}-1}$ (h) $y = \dfrac{10}{x^2-9}$ (i) $y = \ln \dfrac{x+2}{x+3}$

Řešení: (a) $(-\infty, 1\rangle$, (b) $\mathbf{R}$, (c) $(-1, 2)$, (d) $(-\infty, 2)$, (e) $(-\infty, -3\rangle \cup \langle -1, +\infty)$, (f) $(-\infty, -1) \cup (-1, 0) \cup (0, +\infty)$, (g) $\langle 0, 1) \cup (1, +\infty)$, (h) $\mathbf{R} - \{-3, 3\}$, (i) $(-\infty, -3) \cup (-2, +\infty)$.

Určete definiční obor funkce

a) $f(x) = \sqrt{\log x}$, $[\langle 1, \infty)]$

b) $f(x) = \sqrt{\log(\log x)}$, $[\langle 10, \infty)]$

c) $f(x) = \frac{\sqrt{x}}{\sqrt{6-5x}}$, $[\langle 0, \frac{6}{5})]$

d) $f(x) = \sqrt{(2x-1)(x+3)}$, $[(-\infty, -3\rangle \cup \langle \frac{1}{2}, \infty)]$

e) $f(x) = \frac{1}{\sqrt{2x^2+5x-3}}$, $[(-\infty, -3) \cup (\frac{1}{2}, \infty)]$

f) $f)x) = \sqrt{\frac{x-1}{|x-1|}}$, $[(1, \infty)]$

g) $f(x) = \sqrt{\frac{2}{x+3} + \frac{6}{1-3x} - \frac{5}{3x+2}}$, $[(-3, -\frac{2}{3}) \cup (-\frac{2}{3}, \frac{1}{3})]$

h) $f(x) = \frac{3}{\log\sqrt{\frac{2x+1}{4-x}}}$, $[(-\frac{1}{2}, 1) \cup (1, 4)]$

i) $f(x) = \log\frac{x^2+x-6}{x^2+4x+3}$, $[(-\infty, -3) \cup (-3, -1) \cup (2, \infty)]$

j) $f(x) = 3^{\sqrt{x^2-5}}$, $[(-\infty, -\sqrt{5}\rangle \cup \langle\sqrt{5}, \infty)]$