Vypočítejte dané integrály

1) $\int x^2 \cos x dx$

2) $\int x e^{2x} dx$

3) $\int \arcsin x dx$

4) $\int x arctg x dx$

5) $\int x^2 \ln x dx$

6) $\int (1+x)e^x dx$

7) $\int \cos(\ln x) dx$

8) $\int \frac{x^2}{\sqrt[3]{x^3-2}} dx$

9) $\int (\sin x + 7)^3 \cos x dx$

10) $\int \frac{\sin x}{1+\cos^2 x} dx$

11) $\int x e^{x^2-4} dx$

12) $\int tg^3 x dx$

13) $\int \sin^2 x \cos x dx$

14) $\int x\sqrt{x^2-5} dx$

15) $\int \frac{\sin x}{\sqrt[3]{1+4\cos x}} dx$

16) $\int \frac{1}{x\sqrt{1-\ln^2 x}} dx$

$$\bullet \int x^2 \cdot \cos x \, dx = \left| \begin{matrix} u = x^2 & u' = 2x \\ v' = \cos x & v = \sin x \end{matrix} \right| =$$

$$= x^2 \sin x - \int 2x \sin x \, dx = \left| \begin{matrix} u = 2x & u' = 2 \\ v' = \sin x & v = -\cos x \end{matrix} \right| =$$

$$= x^2 \cdot \sin x - \left(-2x \cos x - \int -2 \cos x \, dx\right) =$$

$$= x^2 \cdot \sin x + 2x \cos x - 2 \int \cos x \, dx =$$

$$= x^2 \cdot \sin x + 2x \cos x - 2 \sin x + c$$

$$\bullet \int x \cdot e^{2x} dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = e^{2x} & v = \frac{e^{2x}}{2} \end{matrix} \right| =$$

$$= \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$\bullet \int \arcsin x \, dx = \left| \begin{matrix} u = \arcsin x & u' = \frac{1}{\sqrt{1-x^2}} \\ v' = 1 & v = x \end{matrix} \right| =$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx =$$

$\nwarrow$ subst. $t = 1 - x^2$

$dt = -2x \, dx$

$\frac{dt}{-2x} = dx$

$$= x \arcsin x - \int \frac{x}{\sqrt{t}} \cdot \frac{dt}{-2x}$$

$$= x \arcsin x - \int \frac{1}{-2\sqrt{t}} dt =$$

$$= x \arcsin x + \frac{1}{2} \int t^{-\frac{1}{2}} dt = x \arcsin x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} =$$

$$= x \arcsin x + \sqrt{1-x^2} + c$$

$$\bullet \int x \cdot \operatorname{arctg} x \, dx = \left| \begin{matrix} u = \operatorname{arctg} x & u' = \frac{1}{1+x^2} \\ v' = x & v = \frac{x^2}{2} \end{matrix} \right| =$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{x^2+1} dx$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\,dx =$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int 1 - \frac{1}{x^2+1}\,dx =$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\left(x - \operatorname{arctg} x\right) =$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + c$$

- $$\int x^2 \cdot \ln x\,dx = \left| \begin{matrix} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{matrix} \right| =$$

$$= \frac{x^3}{3}\ln x - \int \frac{1}{x}\cdot\frac{x^3}{3}\,dx = \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx =$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + c$$

- $$\int (1+x)\cdot e^x\,dx = \left| \begin{matrix} u = 1+x & u' = 1 \\ v' = e^x & v = e^x \end{matrix} \right| =$$

$$= (1+x)\cdot e^x - \int e^x\,dx = (1+x)\cdot e^x - e^x + c$$

- $$\int \cos(\ln x)\,dx = \left| \begin{matrix} u = \cos(\ln x) & u' = -\sin(\ln x)\cdot\frac{1}{x} \\ v' = 1 & v = x \end{matrix} \right|$$

$$= x\cos(\ln x) - \int -\sin(\ln x)\,dx =$$

$$= x\cos(\ln x) + \int \sin(\ln x)\,dx = \left| \begin{matrix} u = \sin(\ln x) & u' = \cos(\ln x)\cdot\frac{1}{x} \\ v' = 1 & v = x \end{matrix} \right|$$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\,dx$$

dorieším jakorovnici: $\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)$

$$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x)$$

$$\int \cos(\ln x)\,dx = \frac{x\cos(\ln x) + x\sin(\ln x)}{2} + c$$

- $$\int \frac{x^2}{\sqrt[3]{x^3-2}}\,dx = \left| \begin{array}{l} t = x^3-2 \\ dt = 3x^2\,dx \\ \frac{dt}{3x^2} = dx \end{array} \right| =$$

$$= \int \frac{x^2}{\sqrt[3]{t}} \cdot \frac{dt}{3x^2} = \frac{1}{3}\int t^{-\frac{1}{3}}\,dt = \frac{1}{3}\,\frac{t^{\frac{2}{3}}}{\frac{2}{3}} =$$

$$= \frac{1}{3}\cdot\frac{3}{2}\cdot t^{\frac{2}{3}} = \frac{1}{2}\sqrt[3]{(x^3-2)^2} + C$$

- $$\int (\sin x + 7)^3 \cdot \cos x\,dx = \left| \begin{array}{l} t = \sin x + 7 \\ dt = \cos x\,dx \\ \frac{dt}{\cos x} = dx \end{array} \right|$$

$$= \int t^3 \cdot \cos x \cdot \frac{dt}{\cos x} = \int t^3\,dt = \frac{t^4}{4} = \frac{(\sin x + 7)^4}{4} + C$$

- $$\int \frac{\sin x}{1+\cos^2 x}\,dx = \left| \begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ \frac{dt}{-\sin x} = dx \end{array} \right| =$$

$$= \int \frac{\sin x}{1+t^2} \cdot \frac{dt}{-\sin x} = -\int \frac{1}{1+t^2}\,dt = -\operatorname{arctg} t =$$

$$= -\operatorname{arctg}(\cos x) + C$$

- $$\int x \cdot e^{x^2-4}\,dx = \left| \begin{array}{l} t = x^2-4 \\ dt = 2x\,dx \\ \frac{dt}{2x} = dx \end{array} \right| = \int x \cdot e^t \cdot \frac{dt}{2x} =$$

$$= \int \frac{1}{2} e^t\,dt = \frac{1}{2}e^t = \frac{1}{2}e^{x^2-4} + C$$

- $$\int tg^3 x\,dx = \int \frac{\sin^3 x}{\cos^3 x}\,dx = \left| \begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ \frac{dt}{-\sin x} = dx \end{array} \right|$$

$$= \int \frac{\sin^3 x}{t^3} \cdot \frac{dt}{-\sin x} = \int -\frac{\sin^2 x}{t^3} \cdot dt =$$

$$\sin^2 x = 1 - \cos^2 x = 1 - t^2$$

$$= \int -\frac{(1-t^2)}{t^3}\,dt = \int -\frac{1}{t^3} + \frac{t^2}{t^3}\,dt =$$

$$= \int -t^{-3} + t^{-1}\,dt = -\frac{t^{-2}}{-2} + \ln|t| =$$

$$= \frac{1}{2t^2} + \ln|t| = \frac{1}{2\cos^2 x} + \ln|\cos x| + C$$

- $$\int \sin^2 x \cdot \cos x\,dx = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\,dx \\ \frac{dt}{\cos x} = dx \end{array} \right| =$$

$$= \int t^2 \cdot \cos x \cdot \frac{dt}{\cos x} = \frac{t^3}{3} = \frac{\sin^3 x}{3} + C$$

- $$\int x \cdot \sqrt{x^2 - 5}\,dx = \left| \begin{array}{l} t = x^2 - 5 \\ dt = 2x\,dx \\ \frac{dt}{2x} = dx \end{array} \right| = \int x \cdot \sqrt{t} \cdot \frac{dt}{2x} =$$

$$= \frac{1}{2}\int t^{\frac{1}{2}}\,dt = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{2} \cdot \frac{2}{3} \cdot \sqrt{t^3} = \frac{1}{3}\sqrt{(x^2-5)^3} + C$$

- $$\int \frac{\sin x}{\sqrt[3]{1+4\cos x}}\,dx = \left|\begin{array}{l} t = 1+4\cos x \\ dt = -4\sin x\,dx \\ \dfrac{dt}{-4\sin x} = dx \end{array}\right| =$$

$$= \int \frac{\sin x}{\sqrt[3]{t}} \cdot \frac{dt}{-4\sin x} = -\frac{1}{4}\int t^{-\frac{1}{3}}\,dt = -\frac{1}{4}\,\frac{t^{\frac{2}{3}}}{\frac{2}{3}} =$$

$$= -\frac{1}{4}\cdot\frac{3}{2}\sqrt[3]{t^2} = -\frac{3}{8}\sqrt[3]{(1+4\cos x)^2} + c$$

- $$\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \left|\begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \\ x\cdot dt = dx \end{array}\right|$$

$$= \int \frac{1}{x\cdot\sqrt{1-t^2}}\cdot x\,dt = \int \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$= \arcsin t = \arcsin(\ln x) + c$$

- $$\int (2x+1)\cdot(x^2+x+5)^{10}\,dx = \left|\begin{array}{l} t = x^2+x+5 \\ dt = (2x+1)\,dx \\ \dfrac{dt}{2x+1} = dx \end{array}\right|$$

$$\int (2x+1)\cdot t^{10}\cdot\frac{dt}{2x+1} = \int t^{10}\,dt = \frac{t^{11}}{11} = \frac{(x^2+x+5)^{11}}{11} + c$$

- $$\int \frac{\ln^2 x}{x}\,dx = \left|\begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \\ x\,dt = dx \end{array}\right| = \int \frac{t^2}{x}\cdot x\,dt =$$

$$= \int t^2\,dt = \frac{t^3}{3} = \frac{\ln^3 x}{3} + c$$