Písemka I skupina A …….

1) Určete znaménka funkce. (tzn. intervaly, na kterých je funkce kladná resp. záporná)

$$f(x)=\frac{x^5-3x^4+4x^2}{x^2+2x-3}$$

2) Rozložte na parciální zlomky:

$$\frac{3x^2+11x+8}{x^3+4x^2+4x}$$

3) Určete podle Frobeniovy věty počet řešení daných soustav rovnic a dále soustavy neřešte:

a)
$$\begin{aligned} a-b+2c&=3\\ 2a+c&=1\\ -2b+3c&=5 \end{aligned}$$

b)
$$\begin{aligned} a-b+2c&=3\\ 2a+c&=1\\ -2b+3c&=4 \end{aligned}$$

4) Nakreslete graf funkce $f(x)=\arccos(x+1)-\frac{\pi}{2}$. Najděte k funkci *f(x)* funkci inverzní. Určete definiční obor a obor hodnot obou funkcí.

5) Vypočítejte:

a) $$\begin{pmatrix} 1 & 0 & 2\\ -1 & 1 & 1\\ 2 & 1 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0\\ 1 & 1\\ 2 & 1 \end{pmatrix}$$

b) $$\begin{vmatrix} 1 & 3\\ -1 & 4 \end{vmatrix}+\begin{vmatrix} 1 & 0 & 3\\ 1 & -2 & 1\\ -1 & 2 & 1 \end{vmatrix}$$

6) Vypočítejte limity:

$$\lim_{x\to\infty}\frac{x+1}{x^2-6x+9}=$$
$$\lim_{x\to 3}\frac{x+1}{x^2-6x+9}=$$
$$\lim_{x\to 0+}\log_{\sqrt{2}}x=$$
$$\lim_{x\to-\infty}e^{-x}=$$

7) Určete derivace daných funkcí:

a. $f(x)=\sqrt{x}\sin x$

b. $g(x)=\left(arctg\left(2x-1\right)\right)^3$

Písemka I skupina B …….

1) Určete znaménka funkce. (tzn. Intervaly, na kterých je funkce kladná resp. záporná)

$$f(x) = -x^6 + 7x^5 - 15x^4 + 9x^3$$

2) Rozložte na parciální zlomky:

$$\frac{3x^2 - x + 4}{x^3 + 4x}$$

3) Vyřešte soustavu rovnic:

$$\begin{aligned} 2a + 2b - 3c + d &= 3 \\ a + 2b + 4c + 2d &= 5 \\ -a + b - c + d &= 1 \\ a - b + 2c - 2d &= -4 \end{aligned}$$

4) Nakreslete graf funkce $f(x) = -\operatorname{arccotg} x + \frac{\pi}{2}$. Najděte k této funkci *f(x)* funkci inverzní. Určete definiční obor a obor hodnot obou funkcí.

5) Vypočítejte:

a) $$\begin{pmatrix} -1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}^{-1}$$

b) $$\begin{vmatrix} -2 & -3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

6) Vypočítejte limity:

$$\lim_{x \to -2} \frac{2 - \sqrt{x+6}}{x+2} =$$

$$\lim_{x \to 0+} \ln x =$$

$$\lim_{x \to -\infty} \left(\frac{4}{3}\right)^x =$$

7) Určete derivace daných funkcí:

a. $f(x) = \frac{2x-1}{x+2} + 3$

b. $g(x) = \sqrt[3]{\ln x^2}$

A

① $f = \frac{x^2(x^3-3x^2+4)}{(x+3)(x-1)}$

$f(1) = 1-3+4 \neq 0$

$f(-1) = -1-3+4 = 0$

$(x^3-3x^2+4) : (x+1) = x^2-4x+4$

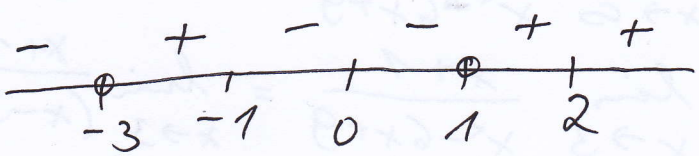
$-(x^3+x^2)$

$-4x^2+4$

$-(-4x^2-4x)$

$4x+4$

$f = \frac{x^2(x+1)(x-2)^2}{(x+3)(x-1)}$



② $\frac{3x^2+11x+8}{x(x^2+4x+4)} = \frac{3x^2+11x+8}{x(x+2)^2} = \frac{A}{x} + \frac{B}{(x+2)^2} + \frac{C}{x+2}$

$3x^2+11x+8 = A(x^2+4x+4) + Bx + C(x^2+2x)$

$3 = A+C$

$11 = 4A+B+2C$

$8 = 4A$

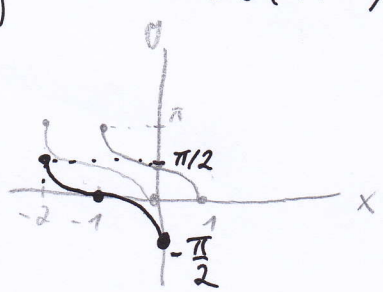
$\left.\begin{matrix} A=2 \\ C=1 \\ B=1 \end{matrix}\right\}$

$\frac{2}{x} + \frac{1}{(x+2)^2} + \frac{1}{x+2}$

③ $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $h(A) = h(A|b) = 2$ soustava.

$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $h(A) \neq h(A|b)$ nemá řešení.

④ $y = \arccos(x+1) - \frac{\pi}{2}$



f^{-1}: $x = \arccos(y+1) - \frac{\pi}{2}$

$x + \frac{\pi}{2} = \arccos(y+1)$

$\cos(x+\frac{\pi}{2}) - 1 = y$

$D_f = \langle -2, 0 \rangle = H_{f^{-1}}$

$D_{f^{-1}} = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle = H_f$

⑤

a) $\begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 2 \\ 5 & 2 \end{pmatrix}$

b) $\begin{vmatrix} 1 & 3 \\ -1 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 3 \\ 1 & -2 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 4+3+(-2)+6+0-6-2-0 =$

$= \underline{\underline{3}}$

⑥ $\lim_{x \to \infty} \frac{x+1}{x^2-6x+9} = 0$

$\lim_{x \to 3} \frac{x+1}{x^2-6x+9} = \lim_{x \to 3} \frac{x+1}{(x-3)^2} = \left[\frac{4}{0}\right] = +\infty$

$\lim_{\substack{x \to 3+ \\ x \to 3-}} \frac{x+1}{(x-3)^2} = \left[\frac{4}{0+}\right] = +\infty$

$\lim_{x \to 0+} \log_{\sqrt{2}} x = -\infty$

$\lim_{x \to -\infty} e^{-x} = \infty$

⑦ $f = \sqrt{x} \cdot \sin x$

$f' = \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$

$g = \operatorname{arctg}^3(2x-1)$

$g' = 3\operatorname{arctg}^2(2x-1) \cdot \frac{1}{1+(2x-1)^2} \cdot 2$

B

① $f = -x^6 + 7x^5 - 15x^4 + 9x^3 = x^3(-x^3 + 7x^2 - 15x + 9)$

$f(1) = -1 + 7 - 15 + 9 = 0$

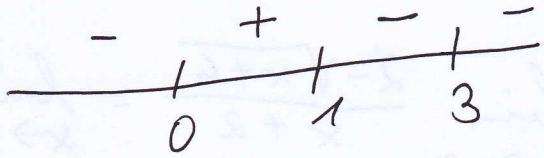
$(-x^3 + 7x^2 - 15x + 9) : (x - 1) = -x^2 + 6x - 9$

$-(-x^3 + x^2)$

$6x^2 - 15x$

$-(6x^2 - 6x)$

$-9x + 9$



$f = -x^3 \cdot (x-1) \cdot (x-3)^2$

② $\frac{3x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$

$3x^2 - x + 4 = Ax^2 + 4A + Bx^2 + C$

$3 = A + B$
$-1 = C$
$4 = 4A$

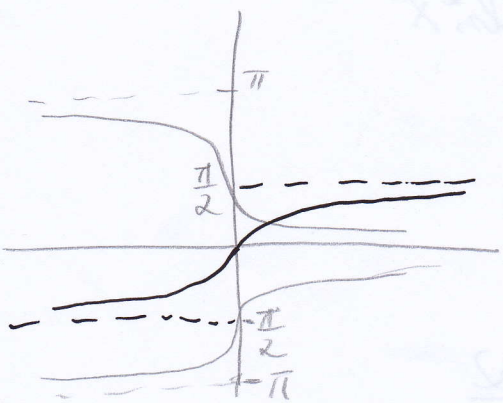
$A = 1$
$B = 2$
$C = -1$

$\frac{1}{x} + \frac{2x - 1}{x^2 + 4}$

③ $\left(\begin{array}{cccc|c} 2 & 2 & -3 & 1 & 3 \\ 1 & 2 & 4 & 2 & 5 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 2 & -2 & -4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 4 & 2 & 5 \\ 0 & 3 & 3 & 3 & 6 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 4 & -5 & 3 & 5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 4 & 2 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & -9 & -1 & -3 \\ 0 & 0 & 0 & -10 & -30 \end{array}\right)$

$d = 3$ $\quad -9c - 3 = -3$ $\quad c = 0$ $\quad b + 0 + 3 = 2$ $\quad b = -1$ $\quad a + (-2) + 6 = 5$ $\quad a = 1$

④ $y = -\operatorname{arccotg} x + \frac{\pi}{2}$



$f^{-1}: \quad x = -\operatorname{arccotg} y + \frac{\pi}{2}$

$-x + \frac{\pi}{2} = \operatorname{arccotg} y$

$\operatorname{cotg}\left(-x + \frac{\pi}{2}\right) = y$

$Df = R = Hf^{-1}$

$Df^{-1} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = Hf$

⑤ a) $\begin{pmatrix} -1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ 5 & -2 \end{pmatrix}$

$\begin{pmatrix} 7 & -2 \\ 5 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 8 & 2 \\ 5 & -3 \end{pmatrix}$

$\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & -1 \end{array}\right)$

⑥ $\lim_{x \to -2} \frac{2-\sqrt{x+6}}{x+2} = \lim_{x \to -2} \frac{2-\sqrt{x+6}}{x+2} \cdot \frac{2+\sqrt{x+6}}{2+\sqrt{x+6}} =$

$= \lim_{x \to -2} \frac{4-x-6}{(x+2)(2+\sqrt{x+6})} = \lim_{x \to -2} \frac{-(x+2)}{(x+2)(2+\sqrt{x+6})} = -\frac{1}{4}$

$\lim_{x \to 0+} \ln x = -\infty$

$\lim_{x \to -\infty} \left(\frac{4}{3}\right)^x = 0$

⑦ $f = \frac{2x-1}{x+2} + 3$

$f' = \frac{2(x+2) - (2x-1)}{(x+2)^2} = \frac{5}{(x+2)^2}$

$g = \sqrt[3]{\ln x^2}$

$g' = \frac{1}{3\sqrt[3]{\ln^2 x^2}} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{3x\sqrt[3]{\ln^2 x^2}}$

⑤ b) $\begin{vmatrix} -2 & -3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$

$-4+3+2-2+0-0-1-0 = \underline{\underline{-2}}$