Vypočítejte integrály:

- $\int e^{3x} - \cos x + \frac{2}{x} + \frac{3}{2-x} - \frac{\cos x}{\sin x - 4}\,dx = \frac{e^{3x}}{3} - \sin x + 2\ln|x| - 3\ln|2-x| - \ln|\sin x - 4| + c$

- $\int \sqrt[3]{x^2} - \frac{1}{\sqrt{x}} + \underbrace{\frac{\sqrt[3]{x}}{\sqrt{x}}}_{= x^{\frac{1}{3}-\frac{1}{2}}} + 2\,dx = \int x^{\frac{2}{3}} - x^{-\frac{1}{2}} + x^{-\frac{1}{6}} + 2\,dx =$
$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{6}}}{\frac{5}{6}} + 2x = \frac{3\sqrt[3]{x^5}}{5} - 2\sqrt{x} + \frac{6\sqrt[6]{x^5}}{5} + 2x + c$

- $\int \frac{2x-5}{x^2} + (x-4)^2 + 3\,dx = \int \frac{2}{x} - \frac{5}{x^2} + x^2 - 8x + 16 + 3\,dx =$
$= \int \frac{2}{x} - 5x^{-2} + x^2 - 8x + 19\,dx = 2\ln|x| - \frac{5x^{-1}}{-1} + \frac{x^3}{3} - \frac{8x^2}{2} + 19x = 2\ln|x| + \frac{5}{x} + \frac{x^3}{3} - 4x^2 + 19x + c$

- $\int \frac{2}{1+x^2} - \frac{x}{x^2+1} + \frac{3}{2x-1}\,dx = \int 2\cdot\frac{1}{1+x^2} - \frac{1}{2}\cdot\frac{2x}{x^2+1} + \frac{3}{2x-1}\,dx =$
$= 2\,\mathrm{arctg}\,x - \frac{1}{2}\ln|x^2+1| + \frac{3\ln|2x-1|}{2} + c$

- $\int 2\cos x - 3\sin x + \frac{2-\sin^3 x}{\sin^2 x}\,dx = \int 2\cos x - 3\sin x + \frac{2}{\sin^2 x} - \sin x\,dx$
$= 2\sin x + 3\cos x - 2\,\mathrm{cotg}\,x + \cos x + c$

- $\int e^{-x} + 3x\cdot\sqrt{x} - \frac{1}{x^2} - 1\,dx = \int e^{-x} + 3x^{\frac{3}{2}} - x^{-2} - 1\,dx =$
$= -e^{-x} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{e^{-1}}{-1} - x = -e^{-x} + \frac{6\sqrt{x^5}}{5} + \frac{1}{e} - x + c$

- $\int \frac{2x+3}{x^2+3x+1} - \frac{2x}{x^2-4} + \frac{2}{\sqrt{1-x^2}} + 3^x\,dx = \ln|x^2+3x+1| - \ln|x^2-4| + 2\arcsin x + \frac{3^x}{\ln 3} + c$

- $\int \frac{1}{x+1} - \frac{1}{x^3} + \frac{1}{2-x} + \frac{x}{x^2-5}\,dx = \int \frac{1}{x+1} - x^{-3} + \frac{1}{2-x} + \frac{1}{2}\cdot\frac{2x}{x^2-5}\,dx =$
$= \ln|x+1| - \frac{x^{-2}}{-2} + \frac{\ln|2-x|}{-1} + \frac{1}{2}\ln|x^2-5| = \ln|x+1| + \frac{1}{2x^2} - \ln|2-x| + \frac{1}{2}\ln|x^2-5| + c$

- $\int x \cdot \sin x \, dx = \left| \begin{matrix} f' = \sin x & f = -\cos x \\ g = x & g' = 1 \end{matrix} \right| = -x\cos x - \int -\cos x \, dx = -x\cos x + \sin x + c$

Per partes $\int f' g = f \cdot g - \int f \cdot g'$

- $\int x^2 \cdot e^{-x} \, dx = \left| \begin{matrix} f' = e^{-x} & f = -e^{-x} \\ g = x^2 & g' = 2x \end{matrix} \right| = -x^2 e^{-x} - \int 2x \cdot (-e^{-x}) \, dx = -x^2 \cdot e^{-x} +$

$+ \int 2x \cdot e^{-x} \, dx = \left| \begin{matrix} f' = e^{-x} & f = -e^{-x} \\ g = 2x & g' = 2 \end{matrix} \right| =$

$= -x^2 e^{-x} - 2x e^{-x} - \int -2e^{-x} \, dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$

- $\int x^2 \cdot \cos x \, dx = \left| \begin{matrix} f' = \cos x & f = \sin x \\ g = x^2 & g' = 2x \end{matrix} \right| = x^2 \cdot \sin x - \int 2x \sin x \, dx =$

$\left| \begin{matrix} f' = \sin x & f = -\cos x \\ g = 2x & g' = 2 \end{matrix} \right| = x^2 \cdot \sin x - \left(-2x\cos x - \int -2\cos x \, dx\right)$

$= x^2 \cdot \sin x + 2x\cos x - 2\sin x + c$

- $\int x \cdot \operatorname{arctg} x \, dx = \left| \begin{matrix} f' = x & f = \frac{x^2}{2} \\ g = \operatorname{arctg} x & g' = \frac{1}{1+x^2} \end{matrix} \right| = \frac{x^2}{2} \operatorname{arctg} x -$

$\int \frac{x^2}{2} \cdot \frac{1}{x^2+1} \, dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \, dx =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} \, dx = \frac{x^2}{2} \operatorname{arctg} x -$

$- \frac{1}{2} \int 1 - \frac{1}{x^2+1} \, dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg} x + c$

- $\int x^3 \cdot \ln x \, dx = \left| \begin{matrix} f' = x^3 & f = \frac{x^4}{4} \\ g = \ln x & g' = \frac{1}{x} \end{matrix} \right| = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx = \frac{x^4}{4} \ln x -$

$- \int \frac{1}{4} \cdot x^3 \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \frac{x^4}{4} = \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$

- $\int x \cdot 2^x \, dx = \left| \begin{matrix} f' = 2^x & f = \frac{2^x}{\ln 2} \\ g = x & g' = 1 \end{matrix} \right| = \frac{x \cdot 2^x}{\ln 2} - \int \frac{2^x}{\ln 2} \, dx = \frac{x \cdot 2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \int 2^x \, dx =$

$= \frac{x \cdot 2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2} + c$

- $\int \cos^2 x \, dx = \left| \begin{matrix} f' = \cos x & f = \sin x \\ g = \cos x & g' = -\sin x \end{matrix} \right| = \sin x \cdot \cos x - \int -\sin^2 x = \sin x \cos x + \int 1 - \cos^2 x \, dx =$

môžeme $\sin^2 x = 1 - \cos^2 x$

$= \sin x \cos x + x - \int \cos^2 x \, dx$

dopočítáme jako rovnici:

$\int \cos^2 x \, dx = \sin x \cos x + x - \int \cos^2 x \, dx$

$2 \int \cos^2 x \, dx = \sin x \cos x + x$

$\int \cos^2 x \, dx = \frac{\sin x \cos x + x}{2} + c$

Per partes $\int u \cdot v' = u \cdot v - \int u' \cdot v$

- $\boxed{\int x^2 \cdot \cos x \, dx} = \left| \begin{matrix} u = x^2 & u' = 2x \\ v' = \cos x & v = \sin x \end{matrix} \right| =$

$= x^2 \sin x - \int 2x \sin x \, dx = \left| \begin{matrix} u = 2x & u' = 2 \\ v' = \sin x & v = -\cos x \end{matrix} \right| =$

$= x^2 \cdot \sin x - \left(-2x \cos x - \int -2 \cos x \, dx \right) =$

$= x^2 \cdot \sin x + 2x \cos x - 2 \int \cos x \, dx =$

$= x^2 \cdot \sin x + 2x \cos x - 2 \sin x + C$

- $\boxed{\int x \cdot e^{2x} \, dx} = \left| \begin{matrix} u = x & u' = 1 \\ v' = e^{2x} & v = \frac{e^{2x}}{2} \end{matrix} \right| =$

$= \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$

- $\boxed{\int \arcsin x \, dx} = \left| \begin{matrix} u = \arcsin x & u' = \frac{1}{\sqrt{1-x^2}} \\ v' = 1 & v = x \end{matrix} \right| =$

$= x \cdot \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx = \left| \begin{matrix} t = 1 - x^2 \\ dt = -2x \, dx \\ \frac{dt}{-2} = x \, dx \end{matrix} \right|$

$= x \cdot \arcsin x - \int \frac{1}{\sqrt{t}} \cdot \frac{dt}{-2} = x \cdot \arcsin x + \frac{1}{2} \int t^{-\frac{1}{2}} dt =$

$= x \arcsin x + \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = x \arcsin x + \sqrt{t} =$

$= x \arcsin x + \sqrt{1-x^2} + C$

- $\boxed{\int x \cdot \operatorname{arctg} x \, dx} = \left| \begin{matrix} u = \operatorname{arctg} x & u' = \frac{1}{1+x^2} \\ v' = x & v = \frac{x^2}{2} \end{matrix} \right| =$

$= \frac{x^2}{2} \operatorname{arctg} x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{x^2+1} dx$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} dx =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int 1 - \frac{1}{x^2+1} dx =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} (x - \operatorname{arctg} x) =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg} x + c$

• $\boxed{\int x^2 \cdot \ln x \, dx} = \left| \begin{matrix} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{matrix} \right| =$

$= \frac{x^3}{3} \ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} dx = \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx =$

$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + c$

• $\boxed{\int (1+x) \cdot e^x dx} = \left| \begin{matrix} u = 1+x & u' = 1 \\ v' = e^x & v = e^x \end{matrix} \right| =$

$= (1+x) \cdot e^x - \int e^x dx = (1+x) \cdot e^x - e^x + c$

• $\boxed{\int \cos(\ln x) dx} = \left| \begin{matrix} u = \cos(\ln x) & u' = -\sin(\ln x) \cdot \frac{1}{x} \\ v' = 1 & v = x \end{matrix} \right|$

$= x \cos(\ln x) - \int -\sin(\ln x) dx =$

$= x \cos(\ln x) + \int \sin(\ln x) dx = \left| \begin{matrix} u = \sin(\ln x) & u' = \cos(\ln x) \cdot \frac{1}{x} \\ v' = 1 & v = x \end{matrix} \right|$

$= x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) dx$

dořeším jako rovnici: $\int \cos(\ln x) dx = x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x)$

$2 \int \cos(\ln x) dx = x \cos(\ln x) + x \sin(\ln x)$

$\int \cos(\ln x) dx = \frac{x \cos(\ln x) + x \sin(\ln x)}{2} + c$

- $\boxed{\int \frac{x^3}{\sqrt[3]{x^3-2}}\,dx} = \left| \begin{array}{l} t = x^3-2 \\ dt = 3x^2\,dx \\ \frac{dt}{3\cancel{x^2}} = dx \cdot x^2 \end{array} \right| =$

$= \int \frac{\cancel{x^2}\,1}{\sqrt[3]{t}} \cdot \frac{dt}{3\cancel{x^2}} = \frac{1}{3}\int t^{-\frac{1}{3}}\,dt = \frac{1}{3}\,\frac{t^{\frac{2}{3}}}{\frac{2}{3}} =$

$= \frac{1}{3} \cdot \frac{3}{2} \cdot t^{\frac{2}{3}} = \frac{1}{2}\sqrt[3]{(x^3-2)^2} + c$

- $\boxed{\int (\sin x + 7)^3 \cdot \cos x\,dx} = \left| \begin{array}{l} t = \sin x + 7 \\ dt = \cos x\,dx \end{array} \right|$

$= \int t^3\,dt \quad = \int t^3\,dt = \frac{t^4}{4} = \frac{(\sin x + 7)^4}{4} + c$

- $\boxed{\int \frac{\sin x}{1+\cos^2 x}\,dx} = \left| \begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ -dt = \sin x\,dx \end{array} \right| =$

$= \int \frac{\cancel{\sin x}\,-dt}{1+t^2} = -\int \frac{1}{1+t^2}\,dt = -\operatorname{arctg} t =$

$= -\operatorname{arctg}(\cos x) + c$

- $\boxed{\int x \cdot e^{x^2-4}\,dx} = \left| \begin{array}{l} t = x^2-4 \\ dt = 2x\,dx \\ \frac{dt}{2\cancel{x}} = x\,dx \end{array} \right| = \int \cancel{x}\,e^t \cdot \frac{dt}{2\cancel{x}} =$

$= \int \frac{1}{2} e^t\,dt = \frac{1}{2} e^t = \frac{1}{2} e^{x^2-4} + c$

- $\boxed{\int tg^3 x\,dx} = \int \frac{\sin^3 x}{\cos^3 x}\,dx = \left|\begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ -dt = dx\cdot\sin x \end{array}\right|$

$= \int \frac{\sin^2 x\cdot \sin x\,dx}{\cos^3 x} = \int \frac{(1-\cos^2 x)\cdot \sin x\,dx}{\cos^3 x} =$

wzorec $\sin^2 x = 1-\cos^2 x$

$= \int -\frac{(1-t^2)}{t^3}\,dt = \int -\frac{1}{t^3} + \frac{t^2}{t^3}\,dt =$

$= \int -t^{-3} + t^{-1}\,dt = -\frac{t^{-2}}{-2} + \ln|t| =$

$= \frac{1}{2t^2} + \ln|t| = \frac{1}{2\cos^2 x} + \ln|\cos x| + C$

- $\boxed{\int \sin^2 x\cdot \cos x\,dx} = \left|\begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array}\right| =$

$= \int t^2\cdot \cancel{\cos x}\,\frac{dt}{\cancel{\cos x}} = \frac{t^3}{3} = \frac{\sin^3 x}{3} + C$

- $\boxed{\int x\cdot\sqrt{x^2-5}\,dx} = \left|\begin{array}{l} t = x^2-5 \\ dt = 2x\,dx \\ \frac{dt}{2\cancel{x}} = dx\cdot x \end{array}\right| = \int \cancel{x}\cdot\sqrt{t}\cdot\frac{dt}{2\cancel{x}} =$

$= \frac{1}{2}\int t^{\frac{1}{2}}\,dt = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{2}\cdot\frac{2}{3}\cdot\sqrt{t^3} = \frac{1}{3}\sqrt{(x^2-5)^3} + C$

$$\bullet \boxed{\int \frac{\sin x}{\sqrt[3]{1+4\cos x}}\,dx} = \left| \begin{array}{l} t = 1+4\cos x \\ dt = -4\sin x\,dx \\ \frac{dt}{-4} = dx \cdot \sin x \end{array} \right| =$$

$$= \int \frac{1}{\sqrt[3]{t}} \cdot \frac{dt}{-4} = -\frac{1}{4}\int t^{-\frac{1}{3}}\,dt = -\frac{1}{4}\,\frac{t^{\frac{2}{3}}}{\frac{2}{3}} =$$

$$= -\frac{1}{4}\cdot\frac{3}{2}\sqrt[3]{t^2} = -\frac{3}{8}\sqrt[3]{(1+4\cos x)^2} + c$$

$$\bullet \boxed{\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx} = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right|$$

$$= \int \frac{1}{\cancel{x}\cdot\sqrt{1-t^2}} \cdot \cancel{x}\,dt = \int \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$= \arcsin t = \arcsin(\ln x) + c$$

$$\bullet \boxed{\int (2x+1)\cdot(x^2+x+5)^{10}\,dx} = \left| \begin{array}{l} t = x^2+x+5 \\ dt = (2x+1)\,dx \end{array} \right|$$

$$\int t^{10}\,dt$$

$$= \int t^{10}\,dt = \frac{t^{11}}{11} = \frac{(x^2+x+5)^{11}}{11} + c$$

$$\bullet \boxed{\int \frac{\ln^2 x}{x}\,dx} = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right| = \int \frac{t^2}{\cancel{x}} \cdot \cancel{x}\,dt =$$

$$= \int t^2\,dt = \frac{t^3}{3} = \frac{\ln^3 x}{3} + c$$

- $\int \frac{1}{4+36x^2}\,dx = \frac{1}{4}\int \frac{1}{1+9x^2}\,dx =$

$= \frac{1}{4}\int \frac{1}{1+(3x)^2}\,dx = \left| \begin{array}{l} t = 3x \\ dt = 3dx \\ \frac{dt}{3} = dx \end{array} \right| = \frac{1}{4}\int \frac{1}{1+t^2}\cdot\frac{dt}{3} =$

$= \frac{1}{12}\int \frac{1}{1+t^2}\,dt = \frac{1}{12}\operatorname{arctg} t = \underline{\frac{1}{12}\operatorname{arctg} 3x + C}$

- $\int \frac{1}{\sqrt{16-49x^2}}\,dx = \int \frac{1}{4\sqrt{1-\frac{49}{16}x^2}}\,dx = \frac{1}{4}\int \frac{1}{\sqrt{1-(\frac{7}{4}x)^2}}\,dx$

$= \frac{1}{4} \left| \begin{array}{l} t = \frac{7}{4}x \\ dt = \frac{7}{4}dx \\ \frac{4}{7}dt = dx \end{array} \right| = \frac{1}{4}\int \frac{1}{\sqrt{1-t^2}}\cdot\frac{4dt}{7} = \frac{1}{7}\arcsin t =$

$= \frac{1}{7}\arcsin\frac{7}{4}x + C$

- $\int \frac{\sin x}{9+\cos^2 x}\,dx = \frac{1}{9}\int \frac{\sin x\,dx}{1+(\frac{\cos x}{3})^2} =$

$= \left| \begin{array}{l} t = \frac{\cos x}{3} \\ dt = -\frac{\sin x}{3}dx \\ -3dt = \sin x\,dx \end{array} \right| = \frac{1}{9}\int \frac{-3\,dt}{1+t^2} = -\frac{1}{3}\operatorname{arctg} t$

$= -\frac{1}{3}\operatorname{arctg}\frac{\cos x}{3} + C$

- $\int \frac{\cos x}{\sqrt{25-\sin^2 x}}\,dx = \int \frac{\cos x\,dx}{5\sqrt{1-(\frac{\sin x}{5})^2}} = \left| \begin{array}{l} t = \frac{\sin x}{5} \\ dt = \frac{\cos x}{5}dx \\ 5dt = \cos x\,dx \end{array} \right|$

$= \int \frac{5dt}{5\sqrt{1-t^2}} = \arcsin t = \arcsin\frac{\sin x}{5} + C$

- $\int \frac{x}{25+4x^2}\,dx \overset{\text{vzorec}}{\underset{\int \frac{f'}{f} = \ln|f|}{=}} \frac{1}{8}\int \frac{8x}{25+4x^2}\,dx = \underline{\frac{1}{8}\ln|25+4x^2| + C}$

- $\int \frac{1}{25+4x^2}\,dx = \frac{1}{25}\int \frac{1}{1+\frac{4}{25}x^2}\,dx = \frac{1}{25}\int \frac{1}{1+(\frac{2}{5}x)^2}\,dx = \left| \begin{array}{l} t = \frac{2}{5}x \\ dt = \frac{2}{5}dx \\ \frac{5}{2}dt = dx \end{array} \right|$

$= \frac{1}{25}\int \frac{1}{1+t^2}\cdot\frac{5}{2}dt = \frac{1}{10}\operatorname{arctg}\frac{2}{5}x + C$

- $\int tg\,x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{-\sin x}{\cos x}dx = -\ln|\cos x| + c$

  recorc $\int \frac{f'}{f} = \ln|f|$

- $\int cotg\,x\,dx = \int \frac{\cos x}{\sin x}dx = \ln|\sin x| + c$

- $\int \ln x\,dx =$ per partes $\int f\cdot g' = f\cdot g - \int f'\cdot g$ $\left| \begin{matrix} f = \ln x & g' = 1 \\ f' = \frac{1}{x} & g = x \end{matrix} \right| =$

  $= x\ln x - \int \frac{1}{x}\cdot x\,dx = x\ln x - \int 1\,dx = x\ln x - x + c$

- $\int \arccos x\,dx = \left| \begin{matrix} f = \arccos x & g' = 1 \\ f' = \frac{-1}{\sqrt{1-x^2}} & g = x \end{matrix} \right| =$

  $= x\arccos x - \int \frac{-x}{\sqrt{1-x^2}}dx = x\arccos x + \int \frac{x\,dx}{\sqrt{1-x^2}}$

  $= \left| \begin{matrix} t = 1-x^2 \\ dt = -2x\,dx \\ \frac{dt}{-2} = x\,dx \end{matrix} \right| = x\arccos x + \int \frac{dt}{-2\cdot\sqrt{t}} =$

  $= x\arccos x - \frac{1}{2}\int t^{-\frac{1}{2}}dt = x\arccos x - \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} =$

  $= x\arccos x - \sqrt{1-x^2} + c$

- $\int arctg\,x\,dx = \left| \begin{matrix} f = arctg\,x & g' = 1 \\ f' = \frac{1}{1+x^2} & g = x \end{matrix} \right| =$

  $= x\,arctg\,x - \int \frac{x}{1+x^2}dx = x\,arctg\,x - \frac{1}{2}\int \frac{2x\,dx}{1+x^2}$

  $= x\,arctg\,x - \frac{1}{2}\ln|1+x^2| + c$

- $\int arccotg\,2x\,dx = \left| \begin{matrix} f = arccotg\,2x & g' = 1 \\ f' = \frac{-1}{1+4x^2}\cdot(2) & g = x \end{matrix} \right| =$

  $= x\,arccotg\,2x - \int \frac{-2x}{1+4x^2}dx = x\,arccotg\,2x + \int \frac{2x}{1+4x^2}dx$

  $= x\,arccotg\,2x + \frac{1}{4}\int \frac{8x}{1+4x^2}dx = x\,arccotg\,2x + \frac{1}{4}\ln|1+4x^2| + c$

- $\int \frac{1}{x} - \frac{2}{x+3} + \frac{3}{2-x}\,dx = \ln|x| - 2\ln|x+3| - 3\ln|2-x| + c$

- $\int \frac{x-1}{x+2}\,dx = \int \frac{x-1+2-2}{x+2}\,dx = \int \frac{x+2}{x+2} - \frac{3}{x+2}\,dx$

$= \int 1 - \frac{3}{x+2}\,dx = x - 3\ln|x+2| + c$

- $\int \frac{3x+1}{x^2+3x-10}\,dx$

$\to D = 9+40 > 0 \to$ parc. zlomky

$$\frac{3x+1}{x^2+3x-10} = \frac{A}{x-2} + \frac{B}{x+5}$$

$3x+1 = Ax+5A+Bx-2B$

$x: \quad 3 = A+B$

$č: \quad 1 = 5A-2B$

$A=1 \quad B=2$

$= \int \frac{1}{x-2} + \frac{2}{x+5}\,dx = \ln|x-2| + 2\ln|x+5| + c$

- $\int \frac{2}{x^2-2x+\cancel{8}\,17}\,dx = \int \frac{2}{(x-1)^2+16}\,dx = \frac{2}{16}\int \frac{1}{\frac{(x-1)^2}{16}+1}\,dx$

$D = 4-\cancel{6}8 < 0 \to$ na arctg

$= \frac{1}{8}\int \frac{1}{\left(\frac{x-1}{4}\right)^2+1}\,dx = \left| \begin{array}{l} t = \frac{x-1}{4} \\ dt = \frac{1}{4}dx \\ 4dt = dx \end{array} \right| = \frac{1}{8}\int \frac{1}{t^2+1}\cdot 4dt =$

$= \frac{1}{2}\,\mathrm{arctg}\,t = \frac{1}{2}\,\mathrm{arctg}\,\frac{x-1}{4} + c$

- $\int \frac{x+15}{x^2+5x}\,dx = \int \frac{3}{x} - \frac{2}{x+5}\,dx = 3\ln|x| - 2\ln|x+5| + c$

$\to D > 0 \to$ parc. zl.

$$\frac{x+15}{x^2+5x} = \frac{A}{x} + \frac{B}{x+5}$$

$x+15 = Ax+5A+Bx$

$x: \quad 1 = A+B$

$č: \quad 15 = 5A$

$A=3 \quad B=-2$

- $$\int \frac{1}{x^2-6x+18}\,dx = \int \frac{1}{(x-3)^2+9}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{x-3}{3}\right)^2+1}\,dx$$

$D<0$

$$= \left| \begin{array}{l} t = \frac{x-3}{3} \\ dt = \frac{1}{3}dx \\ 3dt = dx \end{array} \right| = \frac{1}{9}\int \frac{1}{t^2+1}\cdot 3dt = \frac{1}{3}\operatorname{arctg}\frac{x-3}{3} + c$$

- $$\int \frac{x-1}{x^2+4x+13} = \frac{1}{2}\int \frac{2x-2+4-4}{x^2+4x+13}\,dx =$$

$D<0$, v čitateli je $x \Rightarrow$ nejdřív $\int \frac{f'}{f}$ a pak arctg.

$f' = 2x+4$

$$= \frac{1}{2}\int \frac{2x+4}{x^2+4x+13} - \frac{6}{x^2+4x+13}\,dx =$$

$$= \frac{1}{2}\ln|x^2+4x+13| - \frac{1}{2}\int \frac{6}{(x+2)^2+9}\,dx =$$

$$= \frac{1}{2}\ln|x^2+4x+13| - \frac{3}{9}\int \frac{1}{\left(\frac{x+2}{3}\right)^2+1}\,dx = \left| \begin{array}{l} t = \frac{x+2}{3} \\ dt = \frac{1}{3}dx \\ 3dt = dx \end{array} \right|$$

$$= \frac{1}{2}\ln|x^2+4x+13| - \frac{1}{3}\int \frac{1}{t^2+1}\cdot 3dt =$$

$$= \frac{1}{2}\ln|x^2+4x+13| - \operatorname{arctg}\frac{x+2}{3} + c$$

- $$\int \frac{x^2-1}{x^2+1}\,dx = \text{vydělit}$$

$$(x^2-1):(x^2+1) = 1 - \frac{2}{x^2+1}$$
$$\underline{-(x^2+1)}$$
$$-2$$

$$= \int 1 - \frac{2}{x^2+1}\,dx = x - 2\operatorname{arctg} x + c$$

- $$\int \frac{x^2+1}{x^2-1}\,dx = \text{vydělit} \quad (x^2+1):(x^2-1) = 1 + \frac{2}{x^2-1}$$
$$\underline{-(x^2-1)}$$
$$2$$

$$= \int 1 + \frac{2}{x^2-1}\,dx = \int 1 + \frac{1}{x-1} - \frac{1}{x+1}\,dx = x + \ln|x-1| - \ln|x+1| + c$$

$\rightarrow$ parc. zl.

$$\frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$\vdots$

$A = 1 \qquad B = -1$

- $\displaystyle\int \frac{x^3+x^2-7x+10}{x^2+3x-4}\,dx$ → v čitateli větší mocnina ⇒ vydělit

$$(x^3+x^2-7x+10):(x^2+3x-4) = x-2 + \frac{3x+2}{x^2+3x-4}$$
$$\underline{-(x^3+3x^2-4x)}$$
$$-2x^2-3x+10$$
$$\underline{-(-2x^2-6x+8)}$$
$$3x+2$$

$$= \int x-2+\frac{3x+2}{x^2+3x-4}\,dx = \int x-2+\frac{1}{x-1}+\frac{2}{x+4}\,dx =$$

$D>0$ → parc. zl.

$$= \underline{\frac{x^2}{2}-2x+\ln|x-1|+2\ln|x+4|+c}$$

$$\frac{3x+2}{x^2+3x-4} = \frac{A}{x-1}+\frac{B}{x+4}$$
$$3x+2 = Ax+4A+Bx-B$$
x: $3 = A+B$

c: $2 = 4A-B$

$A=1 \quad B=2$

- $\displaystyle\int \frac{x^2}{x^2+8x+20}\,dx =$ vydělit

$$x^2:(x^2+8x+20) = 1+\frac{-8x-20}{x^2+8x+20}$$
$$\underline{-(x^2+8x+20)}$$
$$-8x-20$$

$$= \int 1-\frac{8x+20}{x^2+8x+20}\,dx = \int 1-4\cdot\frac{2x+5+8-8}{x^2+8x+20}\,dx =$$

→ $D<0$ → $\frac{f'}{f}$ + arctg

$$= x-4\cdot\int\frac{2x+8}{x^2+8x+20}-\frac{3}{x^2+8x+20}\,dx = x-4\ln|x^2+8x+20|$$

$$+4\cdot\int\frac{3}{(x+4)^2+4}\,dx = x-4\ln|x^2+8x+20|+\frac{12}{4}\int\frac{1}{\left(\frac{x+4}{2}\right)^2+1}\,dx$$

$$= \left|\begin{array}{l} t=\frac{x+4}{2} \\ dt=\frac{1}{2}dx \\ 2dt=dx \end{array}\right| = x-4\ln|x^2+8x+20|+3\int\frac{1}{t^2+1}\cdot 2dt =$$

$$= x-4\ln|x^2+8x+20|+6\,\mathrm{arctg}\,\frac{x+4}{2}+c$$