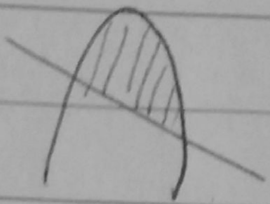
A

1) $y = 3 - x^2$
$y = 1 - x$



$3 - x^2 = 1 - x$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$
$x = 2 \quad x = -1$

$$S = \int_{-1}^{2} 3 - x^2 - (1-x)\,dx = \int_{-1}^{2} 2 - x^2 + x\,dx =$$

$$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2} = 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right) =$$

$$= 6 - \frac{8}{3} + 2 - \frac{1}{3} - \frac{1}{2} = \underline{4{,}5}$$

2) $y' - 3y = 0$

$\lambda - 3 = 0$

$\lambda = 3$

$y = c \cdot e^{3x}$

nebo
$\frac{dy}{dx} = 3y$
$\int \frac{dy}{y} = \int 3\,dx$

variace konst:

$y = c \cdot e^{3x}$

$y' = c' \cdot e^{3x} + c \cdot e^{3x} \cdot 3$

dosadím do zadání

$$c' \cdot e^{3x} + c \cdot e^{3x} \cdot 3 - 3c \cdot e^{3x} = \frac{1 + 3x^3}{x^2} \cdot e^{3x}$$

$$c' = \frac{1 + 3x^3}{x^2}$$

$$c = \int \frac{1}{x^2} + 3x\,dx$$

$$c = \frac{x^{-1}}{-1} + \frac{3x^2}{2} = -\frac{1}{x} + \frac{3x^2}{2} + c$$

$$\underline{\underline{y = \left(-\frac{1}{x} + \frac{3x^2}{2} + c\right) \cdot e^{3x}}}$$

3) $y'' + 3y' = 0$

I. $r^2 + 3r = 0$

$r(r+3) = 0$

$r = 0 \quad r = -3$

$y = C_1 + C_2 e^{-3x}$

II. $y = (ax^2 + bx + c)\cdot x$

$= ax^3 + bx^2 + cx$

$y' = 3ax^2 + 2bx + c$

$y'' = 6ax + 2b$

$6ax + 2b + 9ax^2 + 6bx + 3c = 9x^2 + 10$

$x^2: \quad 9a = 9 \qquad a = 1$

$x: \quad 6a + 6b = 0 \qquad b = -1$

$\check{c}: \quad 2b + 3c = 10 \qquad c = 4$

$y = x^3 - x^2 + 4x$

$y = C_1 + C_2 e^{-3x} + x^3 - x^2 + 4x$

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4) $\int \frac{-6x+12}{x^2-2x+10}\,dx \qquad D = 4 - 4\cdot 10 < 0$

$\int \frac{-3\cdot(2x-4)}{x^2-2x+10}\,dx = -3\int \frac{2x-4-2+2}{x^2-2x+10}\,dx$

$= -3\cdot\int \frac{2x-2}{x^2-2x+10} - \frac{2}{x^2-2x+10}\,dx =$

$= -3\ln|x^2-2x+10| + 6\int \frac{1}{(x-1)^2+9}\,dx =$

$$= -3\ln|x^2-2x+10| + \frac{6}{9}\cdot\int \frac{1}{\left(\frac{x-1}{3}\right)^2+1}\,dx$$

$$= \left| \begin{array}{l} t = \frac{x-1}{3} \\ dt = \frac{1}{3}dx \end{array} \right| = -3\ln|x^2-2x+10|$$

$$+ \frac{2}{3}\cdot\frac{1}{\frac{1}{3}}\cdot\int \frac{\frac{1}{3}}{\left(\frac{x-1}{3}\right)^2+1}\,dx =$$

$$= -3\ln|x^2-2x+10| + \frac{2}{3}\cdot 3\cdot\int \frac{dt}{t^2+1} =$$

$$= \underline{\underline{-3\ln|x^2-2x+10| + 2\operatorname{arctg}\frac{x-1}{3} + C}}$$

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5) $$\int_e^{\infty} \frac{1}{x\ln x}\,dx = \lim_{c\to\infty} \int_e^{c} \frac{1}{x\ln x}\,dx = *$$

$$\int \frac{1}{x\cdot\ln x}\,dx = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}dx \end{array} \right| = \int \frac{dt}{t} = \ln|t| = \ln|\ln x| + C$$

$$* = \lim_{c\to\infty} \Big[\ln|\ln x|\Big]_e^c = \lim_{c\to\infty} \ln|\ln c| - \ln|\ln e| =$$

$$= \infty - 0 = \underline{\underline{\infty}}$$

6) $f = 2x^3 - 3xy + 2y^3 + 1$

$f'_x = 6x^2 - 3y$ $\qquad 6x^2 - 3y = 0 \quad /:3$

$f'_y = -3x + 6y^2$ $\qquad -3x + 6y^2 = 0 \quad /:3$

$2x^2 = y \leftarrow 2x^2 - y = 0$

$\rightarrow -x + 2y^2 = 0$

$-x + 2\cdot(2x^2)^2 = 0$

$A[0; 0]$ $\qquad -x + 8x^4 = 0$

$B[\frac{1}{2}; \frac{1}{2}]$ $\qquad x(-1 + 8x^3) = 0$

$x = 0 \qquad x = \frac{1}{2}$

$$D_2 = \begin{vmatrix} 12x & -3 \\ -3 & 12y \end{vmatrix}$$

$A: D_2 = \begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix} = -9$ není extrém (sedlo)

$B: D_2 = \begin{vmatrix} 6 & -3 \\ -3 & 6 \end{vmatrix} = 27$ lok. MIN

Ⓑ

1) $V = \pi \cdot \int_0^1 \left(\sqrt{x} \cdot e^{-2x}\right)^2 dx$

$= \pi \cdot \int_0^1 x \cdot e^{-4x}\, dx = \left| \begin{matrix} u = x & v' = e^{-4x} \\ u' = 1 & v = -\frac{1}{4} e^{-4x} \end{matrix} \right|$

$\int e^{-4x} dx = \left| \begin{matrix} t = -4x \\ dt = -4dx \end{matrix} \right|$

$= -\frac{1}{4} \int -4 e^{-4x} dx = -\frac{1}{4} \int e^t dt$

$= -\frac{1}{4} e^t = -\frac{1}{4} e^{-4x} + C$

$= \pi \cdot \left(\left[-\frac{1}{4} x e^{-4x} \right]_0^1 - \int_0^1 -\frac{1}{4} e^{-4x} dx \right) =$

$= \pi \cdot \left(-\frac{1}{4} e^{-4} + 0 + \frac{1}{4} \int_0^1 e^{-4x} dx \right) =$

$= \pi \left(-\frac{1}{4} e^{-4} + \frac{1}{4} \cdot \left[-\frac{1}{4} e^{-4x} \right]_0^1 \right) =$

$= \pi \left(-\frac{1}{4} e^{-4} + \frac{1}{4} \left(-\frac{1}{4} e^{-4} + \frac{1}{4} e^0 \right) \right) =$

$= \pi \left(-\frac{1}{4} e^{-4} - \frac{1}{16} e^{-4} + \frac{1}{16} \right) = \underline{\underline{\frac{\pi}{16} \left(-5 e^{-4} + 1 \right)}}$

2) $y' + y \cdot \operatorname{tg} x = 0$

$\frac{dy}{dx} = -y \operatorname{tg} x$

$$\int \frac{dy}{y} = \int -tg\,x\,dx$$

$$\ln|y| = \int -\frac{\sin x}{\cos x}\,dx$$

$$\ln|y| = \ln|\cos x| + c$$

$$\ln|y| = \ln|\cos x| + \ln c$$

$$y = \cos x \cdot c$$

variace konst.

$y = \cos x \cdot c \qquad y' = -\sin x \cdot c + \cos x \cdot c'$

dosadím do zadání -

$$-c\sin x + c'\cos x + \cancel{\cos x} \cdot c \cdot \frac{\sin x}{\cancel{\cos x}} = \cos^2 x$$

$$c'\cos x = \cos^2 x$$

$$c' = \cos x$$

$$c = \int \cos x\,dx$$

$$c = \sin x + c$$

$$\underline{y = \cos x \cdot (\sin x + c)}$$

3) $y'' - 5y' - 6y = 0$

I. $r^2 - 5r - 6 = 0$

$(r+1)(r-6) = 0$

$r = -1 \quad r = 6$

$y = c_1 e^{-x} + c_2 e^{6x}$

II. $y = a \cdot e^{6x} \cdot x$

$y' = ae^{6x} \cdot 6 \cdot x + ae^{6x}$

$y'' = ae^{6x} \cdot 36 \cdot x + ae^{6x} \cdot 6 + ae^{6x} \cdot 6$

dosadím:

$ae^{6x} \cdot 36x + 6ae^{6x} + 6ae^{6x} - 5ae^{6x} \cdot 6x - 5ae^{6x}$

$- 6ae^{6x} \cdot x = 14e^{6x} \quad / : e^{6x}$

$36ax + 12a - 30ax - 5a - 6ax = 14$

$7a = 14$

$a = 2$

$y = 2xe^{6x}$

$$\underline{\underline{y = C_1 e^{-x} + C_2 e^{6x} + 2xe^{6x}}}$$

4) $(2x^3 + 2x^2 - 5x + 4) : (x^2 + x - 2) = 2x + \frac{-x+4}{x^2+x-2}$

$-\underline{(2x^3 + 2x^2 - 4x)}$

$-x + 4$

$$\int 2x + \frac{-x+4}{x^2+x-2} \, dx$$

$D = 1 + 8 > 0 \Rightarrow$ parc. zlomky

$$\frac{-x+4}{x^2+x-2} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

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$$-x+4=A(x-1)+B(x+2)$$

$$x=1 \rightarrow \quad 3=3B$$

$$B=1$$

$$x=-2 \rightarrow \quad 6=-3A$$

$$A=-2$$

$$\int 2x+\frac{-2}{x+2}+\frac{1}{x-1}\,dx$$

$$=\underline{x^2-2\ln|x+2|+\ln|x-1|+c}$$

5) $$\int_0^\infty \frac{dx}{x^2+2x+2}=\lim_{c\to\infty}\int_0^c \frac{dx}{x^2+2x+2}=*$$

$$\int \frac{dx}{x^2+2x+2}=\int\frac{dx}{(x+1)^2+1}=\left|\begin{array}{l} t=x+1\\ dt=dx\end{array}\right|=$$

$D=4-8<0$

$$=\int\frac{dt}{t^2+1}=\operatorname{arctg} t=\operatorname{arctg}(x+1)+c$$

$$*=\lim_{c\to\infty}\left[\operatorname{arctg}(x+1)\right]_0^c=$$

$$=\lim_{c\to\infty}\underbrace{\operatorname{arctg}(c+1)}_{\frac{\pi}{2}}-\underbrace{\operatorname{arctg}1}_{\frac{\pi}{4}}=$$

$$\frac{\pi}{2}-\frac{\pi}{4}=\underline{\frac{\pi}{4}}$$

6) $f = 6 - 4x - 3y$

$g: \quad x^2 + y^2 = 1 \rightarrow$ Jakobián

$$\begin{vmatrix} -4 & -3 \\ 2x & 2y \end{vmatrix} = 0$$

$$-8y + 6x = 0$$

$$6x = 8y$$

$$x = \frac{4}{3}y \rightarrow x^2 + y^2 = 1$$

A $\left[\frac{4}{5} ; \frac{3}{5}\right]$

B $\left[-\frac{4}{5} ; -\frac{3}{5}\right]$

$$\frac{16}{9}y^2 + y^2 = 1 \quad / \cdot 9$$

$$16y^2 + 9y^2 = 9$$

$$25y^2 = 9$$

$$y = \pm\frac{3}{5}$$

$f(A) = 6 - \frac{16}{5} - \frac{9}{5} = 1$ váz. MIN

$f(B) = 6 + \frac{16}{5} + \frac{9}{5} = 11$ váz. MAX

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